GRAVITY: THE INSIDE STORY

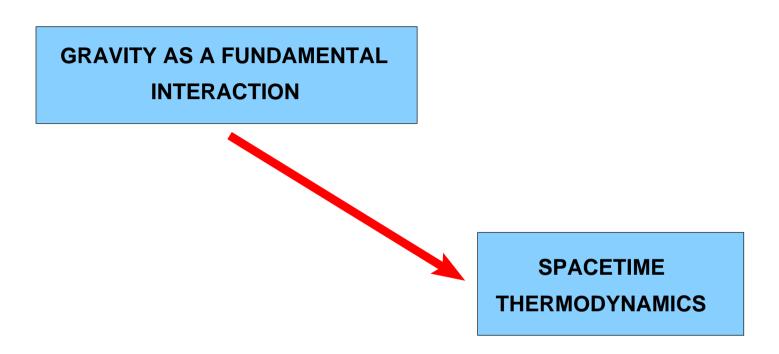
T. Padmanabhan (IUCAA, Pune, India)

VR Lecture, IAGRG Meeting Kolkatta, 28 Jan 09

CONVENTIONAL VIEW

GRAVITY AS A FUNDAMENTAL INTERACTION

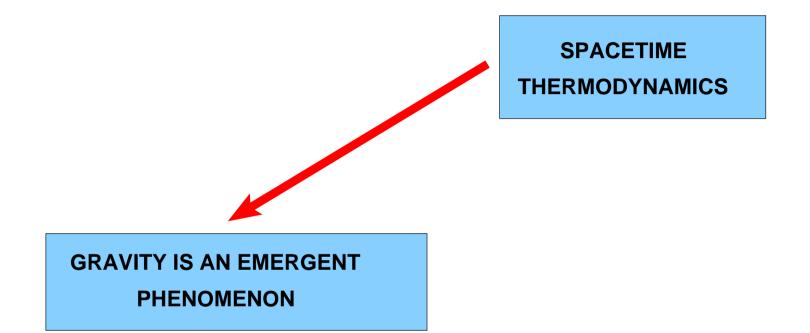
CONVENTIONAL VIEW



NEW PERSPECTIVE

GRAVITY IS AN EMERGENT PHENOMENON

NEW PERSPECTIVE



GRAVITY IS THE THERMODYNAMIC LIMIT OF THE STATISTICAL MECHANICS OF 'ATOMS OF SPACETIME'

SOLIDS

SPACETIME

Mechanics; Elasticity $(\rho, \mathbf{v} \dots)$

Einstein's Theory $(g_{ab} \dots)$

Statistical Mechanics

of atoms/molecules

Statistical mechanics

of "atoms of spacetime"

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Thermodynamics of solids

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- Thermodynamics offers a connection between the two though the form of entropy functional, S[ξ]. No microstructure, no thermodynamics!
- You never took a course in 'quantum thermodynamics'.

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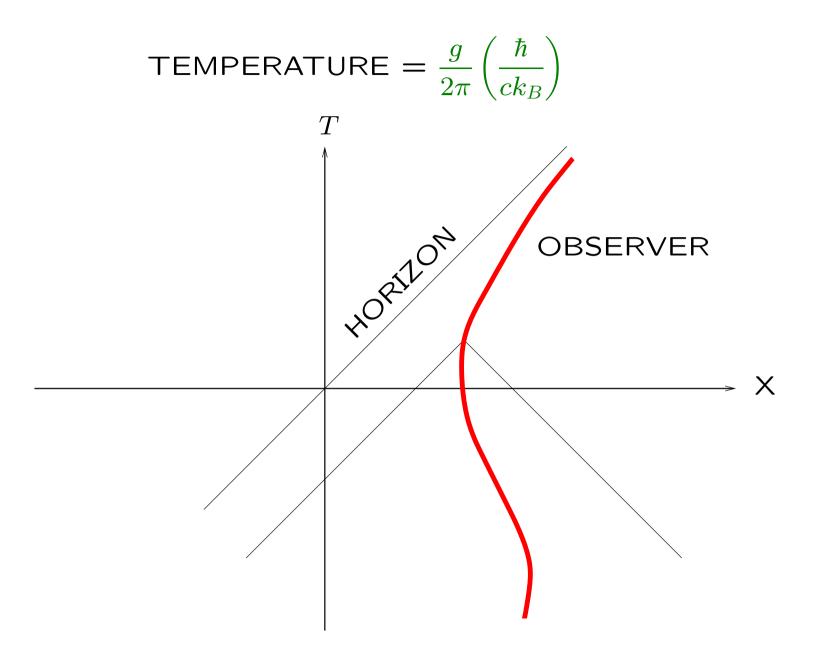
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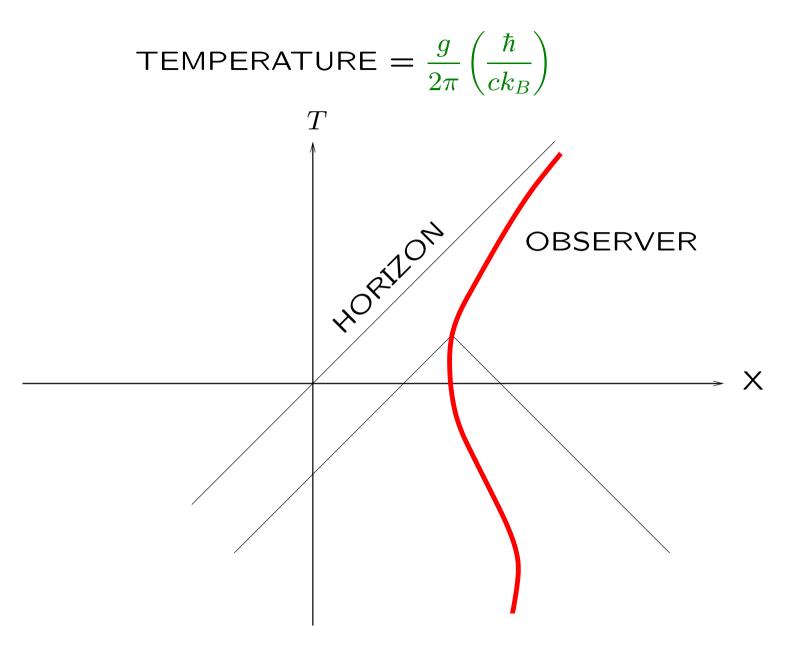
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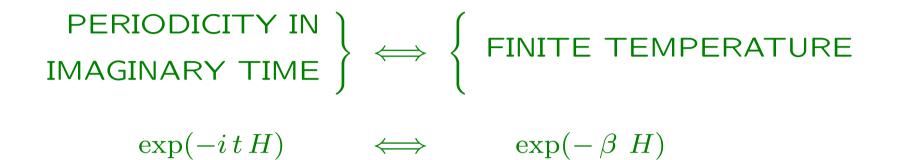




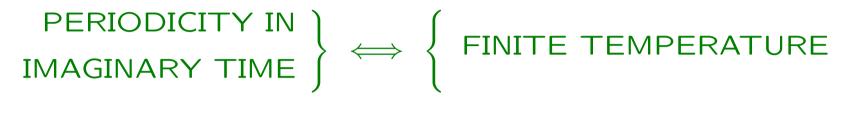
Works for Blackholes, deSitter, Rindler

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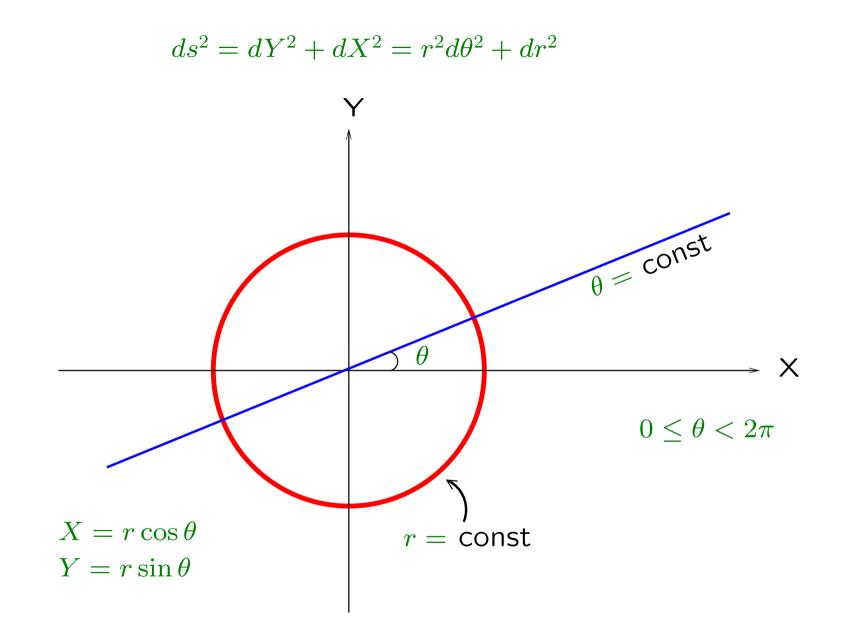


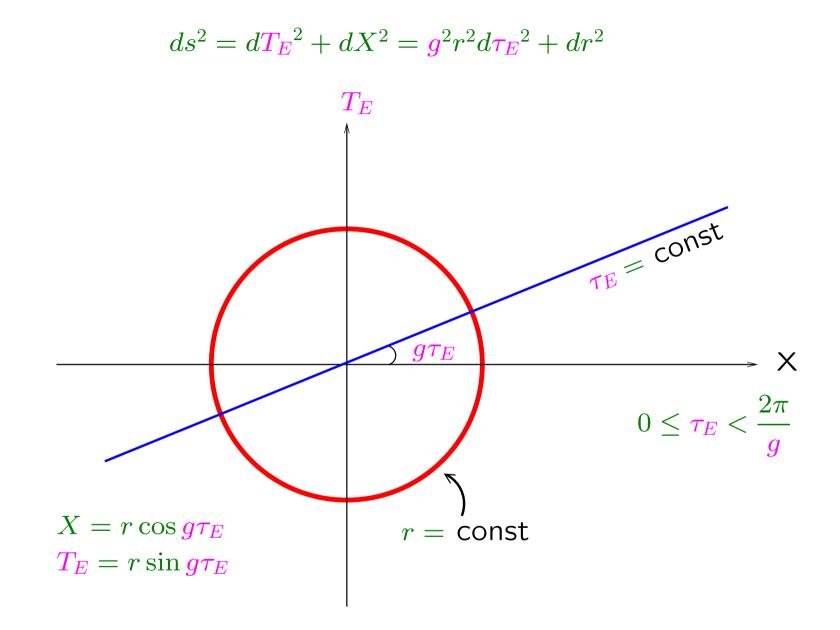
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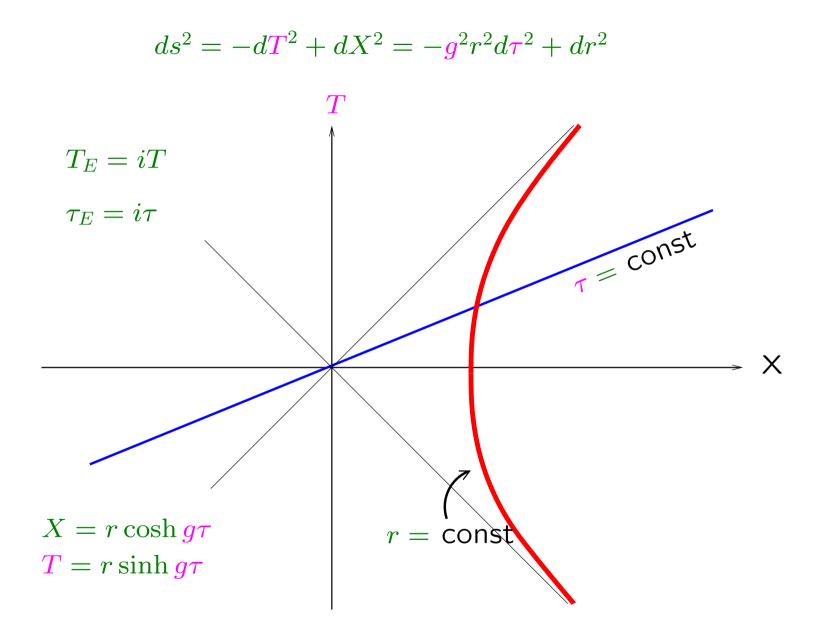


 $\exp(-i\,t\,H) \qquad \iff \qquad \exp(-\,\beta\,\,H)$

SPACETIMES WITH HORIZONS EXHIBIT PERIODICITY IN IMAGINARY TIME \implies TEMPERATURE







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PHYSICS PROGRESSES BY EXPLAINING FEATURES WHICH WE NEVER THOUGHT NEEDED ANY EXPLANATION !! EXAMPLE: $m_{inertial} = m_{grav}$ 1. Why do Einstein's equations reduce to a thermodynamic identity for virtual displacements of horizons ?

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• Read off (with
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[TP, 2002]

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• Works for Kerr, FRW,

[D. Kothawala et al., 06; Rong-Gen Cai, 06, 07]

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[TP, 02, 05]

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• In fact, one can develop a theory with $A_{total} = A_{sur} + A_{matter}$ using the virtual displacements of the horizon as key. [TP, 2005]

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• You find that the part you threw away, the A_{sur} , evaluated on any horizon gives its entropy !

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• The surface term is closely related to horizon entropy in Lanczos-Lovelock theory.

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- Leads to gravity being an emergent phenomenon described by Einstein's equations at lowest order with calculable corrections.

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• Associate with virtual displacements of null surfaces an entropy/ action which is quadratic in deformation field: [T.P, 08; T.P., A.Paranjape, 07]

$$S[\xi] = S[\xi]_{grav} + S_{matt}[\xi]$$

with

$$S_{grav}[\xi] = \int_{\mathcal{V}} d^D x \sqrt{-g} 4 P^{abcd} \nabla_c \xi_a \nabla_d \xi_b; \qquad S_{matt} = \int_{\mathcal{V}} d^D x \sqrt{-g} T^{ab} \xi_a \xi_b$$

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- Demand that the variation should constrain the background.
- This leads to *P^{abcd}* having a (RG-like) derivative expansion in powers of number of derivatives of the metric:

$$P^{abcd}(g_{ij},R_{ijkl}) = c_1 \overset{(1)}{P}^{abcd}(g_{ij}) + c_2 \overset{(2)}{P}^{abcd}(g_{ij},R_{ijkl}) + \cdots,$$

• The *m*-th order term is unique: $\overset{(m)}{P}{}^{abcd} = (\partial \mathcal{L}_{(m)} / \partial R_{abcd});$

A NEW VARIATIONAL PRINCIPLE

 Associate with virtual displacements of null surfaces an entropy/ action which is quadratic in deformation field: [T.P, 08; T.P., A.Paranjape, 07]

$$S[\xi] = S[\xi]_{grav} + S_{matt}[\xi]$$

with

$$S_{grav}[\xi] = \int_{\mathcal{V}} d^D x \sqrt{-g} 4 P^{abcd} \nabla_c \xi_a \nabla_d \xi_b; \qquad S_{matt} = \int_{\mathcal{V}} d^D x \sqrt{-g} T^{ab} \xi_a \xi_b$$

- Demand that the variation should constrain the background.
- This leads to *P^{abcd}* having a (RG-like) derivative expansion in powers of number of derivatives of the metric:

$$P^{abcd}(g_{ij}, R_{ijkl}) = c_1 \overset{(1)}{P}^{abcd}(g_{ij}) + c_2 \overset{(2)}{P}^{abcd}(g_{ij}, R_{ijkl}) + \cdots,$$

• The *m*-th order term is unique: $\overset{(m)}{P}{}^{abcd} = (\partial \mathcal{L}_{(m)} / \partial R_{abcd});$

• Example: The lowest order term is:

$$S_1[\xi] = \int_{\mathcal{V}} rac{d^D x}{8\pi} \left(
abla_a \xi^b
abla_b \xi^a - (
abla_c \xi^c)^2
ight)$$

• Demand that $\delta S = 0$ for variations of all null vectors: This leads to Lanczos-Lovelock theory with an arbitrary cosmological constant:

$$16\pi \left[P_b^{\ ijk} R^a_{\ ijk} - \frac{1}{2} \delta^a_b \mathcal{L}^{(D)}_m \right] = 8\pi T^a_b + \Lambda \delta^a_b,$$

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- On any solution with horizon, it gives the correct Wald entropy:

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 Further for any solution, in a local Rindler frame, the causal horizons have the correct entropy. At the lowest order, it is quarter of transverse area.

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- So after you have "solved" the cosmological constant problem, if someone introduces $L_{matter} \rightarrow L_{matter} \rho$, you are in trouble again!
- The only way out is to have a formalism for gravity which is invariant under $T_{ab} \rightarrow T_{ab} + \rho g_{ab}$.
- All these have nothing to do with observations of accelerated universe! Cosmological constant problem existed earlier and will continue to exist even if all these observations go away!

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- Then cosmological constant problem cannot be solved; that is, gravitational equations cannot be invariant under $T_{ab} \rightarrow T_{ab} \rho_0 g_{ab}$.
- Drop the assumption that g_{ab} is the dynamical variable, identify new degrees of freedom (virtual displacements of null surfaces in spacetime) associate an entropy with them and obtain the dynamics from extremising the entropy.

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- The field equations have a new 'gauge freedom' and has the form:

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- Introduces a new length scale L_H . (Observationally, $L_P/L_H \approx 10^{-60} \approx \exp(-\sqrt{2}\pi^4)$.)
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• The hierarchy:

$$\rho_{\text{vac}} = \left[\underbrace{\frac{1}{L_P^4}}_{\langle x \rangle}, \underbrace{\frac{1}{L_P^4} \left(\frac{L_P}{L_H} \right)^2}_{\langle x^2 \rangle^{1/2}}, \frac{1}{L_P^4} \left(\frac{L_P}{L_H} \right)^4, \cdots \right]$$

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Connection with thermodynamics	Specify the entropy	Specify the entropy
Resulting equation	Classical / Quantum	Einsteins theory with calculable corrections



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- Connects with A_{sur} giving the horizon entropy

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