

The Rainbows of Gravity

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Abstract

We present here a spectrum of developments and predictions in gravitation theory in recent years which appear to be amongst some of the most exciting directions. These include the spacetime singularities, gravitational collapse final states, and the deep cosmic conundrums that the new results on these issues have revealed. Amongst these are the cosmic censorship and the paradox of predictability in the universe, and the possible emerging implications for a quantum theory of gravity. The likely contact with observations and implications for relativistic astrophysics and black hole physics today are indicated.

Keywords: Singularities, Black holes, Gravitational collapse, Cosmology

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I. INTRODUCTION

After Einstein proposed the general theory of relativity describing the gravitational force in terms of spacetime curvatures, the proposed field equations related the spacetime geometry to the matter content of the universe. In general relativity, the universe is modeled as a spacetime, which has mathematically a structure of a four dimensional differentiable manifold. This means that locally the spacetime is always flat, in a sufficiently small region around any point, but on a larger scale it does not have to be so and it can have more rich and varied structure. A two-dimensional example of such a manifold is a sphere, which is flat enough in the vicinity of any single point on its surface, but has a non-zero global curvature.

The earliest solutions found for the field equations were the Schwarzschild metric representing the gravitational field around an isolated body such as a spherically symmetric star, and the Friedmann cosmological models. Both these contained a spacetime singularity where the curvatures and energy densities were infinite and the physical description would then break down. In the Schwarzschild solution such a singularity was present at the center of symmetry $r = 0$ whereas for the Friedmann models it is found at the epoch $t = 0$ which is beginning of the universe and origin of time where the scale factor for the universe vanishes and all objects are crushed to a zero volume due to infinite gravitational tidal forces.

Even though the physical problem posed by the existence of such a strong curvature singularity was realized immediately in these solutions, which turned out to have several important implications towards the experimental verification of the general relativity theory, initially this phenomenon was not taken seriously. It was generally thought that the existence of such a singularity must be a consequence of the very high degree of symmetry imposed on the spacetime while deriving these solutions. Subsequently, the distinction between a genuine spacetime singularity and a mere coordinate singularity became clear and it was realized that the singularity at $r = 2m$ in the Schwarzschild spacetime was only a coordinate singularity which could be removed by a suitable coordinate transformation. It was clear, however, that the genuine curvature singularity at $r = 0$ cannot be removed by any such transformations. The hope was then that when more general solutions are considered with a less degree of symmetry requirements, such singularities will be avoided. This issue was sorted out when a detailed study of the structure of a general spacetime and the associated problem of singularities was taken up by Hawking, Penrose, and Geroch (see for example, Hawking and Ellis, 1973), which showed that singularities are in fact a much more

general phenomena in gravitation theories.

Further to the general relativity theory in 1915, the gravitation physics was a relatively quiet field with few developments till about 1950s. However, the 1960s saw the emergence of new observations in high energy astrophysics, such as quasars and high energy phenomena at the center of galaxies such as extremely energetic jets. These observations, together with important theoretical developments such as studying the global structure of spacetimes and singularities, led to important results in black hole physics and relativistic astrophysics and cosmology.

My purpose here is to indicate and highlight a spectrum of such developments and results which deal with probably some of the most exciting current issues on which useful research in gravitation and cosmology is centered today. This is of course a personal perspective and no claim to completeness is made. However, I hope that what is presented below will paint an interesting view of the landscape of gravity physics and the emerging cosmic frontiers. While doing so, I discuss what I think to be rather interesting results, including some of our work on the final endstates of gravitationally collapse, cosmic censorship, and black holes and naked singularities. Related major cosmic conundrums such as the issue of predictability in the universe are discussed, and observational implications of naked singularities are indicated.

II. SPACETIME SINGULARITIES

As mentioned above, the work in early 1970s in gravitation theories showed that a spacetime will admit singularities within a rather general framework provided it satisfies certain reasonable physical assumptions such as the positivity of energy, a suitable causality assumption and a condition implying strong gravitational fields, such as the existence of trapped surfaces. It thus followed that singularities form a rather general feature of the relativity theory. In fact, these considerations ensure the existence of singularities in other theories of gravity also which are based on a spacetime manifold framework and that satisfy the general conditions such as those stated above.

Therefore the scenario that emerges is, essentially for all classical spacetime theories of gravitation, the occurrence of singularities form an inevitable and integral part of the description of the physical reality. In the vicinity of such a singularity, typically the energy densities, spacetime curvatures, and all other physical quantities would blow up, thus indicating the occurrence of super ultra-dense regions in the universe. The behaviour of such regions may not be governed by the classical theory itself, which may breakdown having predicted the existence of the singularities,

and a quantum gravitational theory would be the likely description of the phenomena created by such singularities.

Firstly, it is to be clarified how to characterize a spacetime singularity. It turns out that it is the notion of geodesic incompleteness that characterizes a singularity in an effective manner for a spacetime and enables their existence to be proved by means of general enough theorems, which involve a consideration of the gravitational focusing caused by the spacetime curvature in congruences of timelike and null geodesics. This turns out to be the main cause of the existence of singularity in the form of non-spacelike incomplete geodesics in spacetime. The issue of physical nature of a spacetime singularity is important. There are many types of singular behaviours possible for a spacetime and some of these could be regarded as mathematical pathologies in the spacetime rather than having any physical significance. This will be especially so if the spacetime curvature and similar other physical quantities remained finite along an incomplete non-spacelike geodesic in the limit of approach to the singularity. A singularity will be physically important when there is a powerful enough curvature growth along singular geodesics, and the physical interpretation and implications of the same are to be considered.

Considering various situations, the occurrence of nonspacelike geodesic incompleteness has been generally agreed upon as the criterion for the existence of a singularity for a spacetime. It is clear that if a spacetime manifold contains incomplete non-spacelike geodesics, there is a definite singular behaviour present in the spacetime. In such a case, a timelike observer or a photon suddenly disappears from the spacetime after a finite amount of proper time or after a finite value of the affine parameter. The singularity theorems which result from an analysis of gravitational focusing and global properties of a spacetime prove this incompleteness property for a wide class of spacetimes under a set of rather general conditions.

The matter fields with positive energy density affect the causality relations in a spacetime and cause focusing in the families of timelike and null trajectories. The essential phenomena that occurs here is that matter focuses the nonspacelike geodesics of the spacetime into pairs of focal points or the conjugate points. The rate of change of volume expansion for a given congruence of timelike geodesics can be written as

$$\frac{d\theta}{d\tau} = -R_{lk}V^lV^k - \frac{1}{3}\theta^2 - 2\sigma^2 + 2\omega^2$$

where, for a given congruence of timelike geodesics, the quantities θ , σ and ω are *expansion*, *shear*, and *rotation* tensors are respectively. The above equation is called the *Raychaudhuri equation*

(Raychaudhuri, 1955) which describes the rate of change of the volume expansion as one moves along the timelike geodesic curves in the congruence. We note that the second and third term on the right-hand side involving θ and σ are positive always. Consider now the term $R_{ij}V^iV^j$. By Einstein equations this can be written as

$$R_{ij}V^iV^j = 8\pi[T_{ij}V^iV^j + \frac{1}{2}T]$$

The term $T_{ij}V^iV^j$ above represents the energy density as measured by a timelike observer with the unit tangent V^i , which is the four-velocity of the observer. For all reasonable classical physical fields this energy density is generally taken as non-negative and it is assumed that for all timelike vectors V^i the following is satisfied

$$T_{ij}V^iV^j \geq 0$$

Such an assumption is called the *weak energy condition*. When a suitable energy condition is satisfied, the Raychaudhuri equation implies that the effect of matter on spacetime curvature causes a focusing effect in the congruence of timelike geodesics due to gravitational attraction. This, in general causes the neighbouring geodesics in the congruence to cross each other to give rise to caustics or conjugate points. This separation between nearby timelike geodesics is governed by what is called the geodesic deviation equation,

$$D^2Z^j = -R^j{}_{kil}V^kZ^iV^l$$

where Z^i is the separation vector between nearby geodesics of the congruence. Solutions of the above equation are called the *Jacobi fields* along a given timelike geodesic.

There are several singularity theorems available which establish the non-spacelike geodesic incompleteness for a spacetime under different sets of conditions and applicable to different physical situations. However, the most general of these is the Hawking–Penrose theorem (Hawking and Penrose, 1970), which is applicable in both the collapse situation and cosmological scenario. The main idea of the proof of such a theorem is, using the causal structure analysis it is shown that there must be maximal length timelike curves between certain pairs of events in the spacetime. Now, a causal geodesic which is both future and past complete must contain pairs of conjugate points if M satisfies the generic condition and an energy condition. This is then used to draw the necessary contradiction to show that M must be non-spacelike geodesically incomplete.

The inevitable existence of spacetime singularities, for wide classes of general models of spacetimes means that the classical gravity necessarily gives rise to regions in the spacetime universe

where the densities and spacetime curvatures would really grow without any bounds, where all other physical parameters also would diverge really.

III. GRAVITATIONAL COLLAPSE

The existence of spacetime singularities in the Einstein gravity, and in all similar spacetime theories of gravitation poses intriguing challenges and fundamental questions in physics as well as cosmology.

Such a phenomenon will basically arise in two physical scenarios in the universe, the first being the cosmology where such a singularity will correspond to the origin of the universe, and secondly whenever locally a large quantity of matter and energy collapses under the force of its own gravity. This later situation will be effectively realized in the gravitational collapse of massive stars in the universe, which collapse and shrink catastrophically under their self-gravity, when the star has exhausted its nuclear fuel within which earlier supplied the internal pressure to halt the in-fall due to gravity. We now discuss this second possibility in some detail.

When a massive star, more than a few solar masses, has exhausted its internal nuclear fuel, it is believed to enter the stage of an endless gravitational collapse without having any final equilibrium state. According to the Einstein theory of gravitation, the star goes on shrinking in its radius, reaching higher and higher densities. What would be the final fate of such an object according to the general theory of relativity? This is one of the central questions in relativistic astrophysics and gravitation theory today. It has been suggested that the ultra-dense object that forms as a result of collapse could be a black hole in the space and time from which not even light rays can escape. Alternatively, if the event horizon of gravity fails to cover the final crunch, it could be a visible singularity which can causally interact with the outside universe and from which emissions of light and matter may be possible.

It is of course reasonably clear that very near such a spacetime singularity, the classical description that predicted it must itself breakdown. The quantum effects associated with gravity are most likely to become dominant in such a regime. These may resolve the classical singularity. However, we have no viable and consistent quantum theory of gravity available as of today despite many serious attempts, and therefore the issue of resolution of singularities as produced by classical gravity remains very much open currently.

An investigation on final fate of collapse is of importance from both the theoretical as well as

observational point of view. At the theoretical level, working out the collapse outcomes in general relativity is crucial to the problem of asymptotic predictability, namely, whether the singularities forming at the end point of collapse will be necessarily covered by the event horizons of gravity. A hypothesis that remains fundamental to the theoretical foundations of black hole physics and its numerous astrophysical applications which have been invoked in past decades (e.g. the area theorem for black holes, laws of black hole thermodynamics, Hawking radiation effect, predictability; and on observational side, accretion of matter by black holes, massive black holes at the center of galaxies etc), is *cosmic censorship* which states the singularities of collapse must be hidden within horizons of gravity. On the other hand, existence of visible or naked singularities would offer a new approach on these issues requiring modification and reformulation of our usual theoretical conception on black holes.

To investigate this issue, dynamical collapse scenarios have been examined in past decade or so for many cases such as clouds composed of dust, radiation, perfect fluids, or also of matter compositions with more general equations of state (for references and details, see e.g. Joshi 2008).

IV. BLACK HOLES

One could consider a gravitationally collapsing spherical massive star. We need to consider the interior solution for the object which will depend on the properties of matter, equation of state, and the physical processes taking place within the stellar interior. However, assuming the matter to be pressureless dust allows to solve the problem analytically, providing many important insights. Here the energy-momentum tensor is given by $T^{ij} = \rho u^i u^j$, and one needs to solve the Einstein equations for the spherically symmetric metric. This determines the metric potentials, and the interior geometry of the collapsing dust ball is given by,

$$ds^2 = -dt^2 + R^2(t) \left[\frac{dr^2}{1-r^2} + r^2 d\Omega^2 \right]$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric on two-sphere. The geometry outside is vacuum Schwarzschild space-time. The interior geometry of the dust cloud matches at the boundary $r = r_b$ with the exterior Schwarzschild space-time.

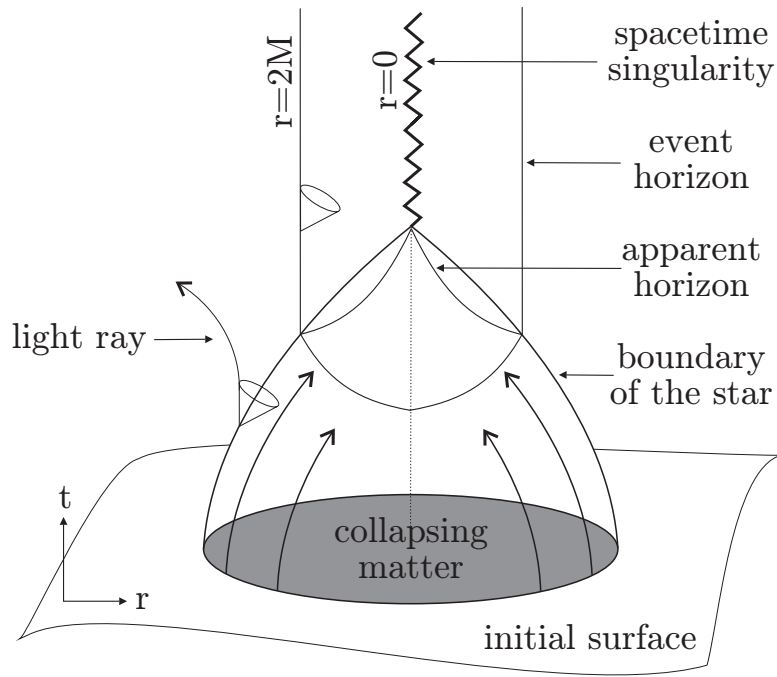


Fig 1. Spherically symmetric homogeneous dust collapse where the final outcome is black hole formation in the spacetime.

The basic features of such a collapsing, spherical, homogeneous dust cloud configuration are given in Fig. 1. The collapse is initiated when the star surface is outside its Schwarzschild radius $r = 2m$, and a light ray emitted from the surface of the star can escape to infinity. However, once the star has collapsed below $r = 2m$, a black hole region of no escape develops in the space-time, bounded by the event horizon at $r = 2m$. Any point in this empty region represents a trapped surface (which is a two-sphere in space-time) in that both the outgoing and ingoing families of null geodesics emitted from this point converge and hence no light comes out of this region. Then, the collapse to an infinite density and curvature singularity at $r = 0$ becomes inevitable in a finite proper time as measured by an observer on the surface of the star. The black hole region in the resulting vacuum Schwarzschild geometry is given by $0 < r < 2m$, the event horizon being the outer boundary. On the event horizon, the radial outwards photons stay where they are, but all the rest are dragged inwards. No information from this black hole can propagate outside $r = 2m$ to observers far away. We thus see that the collapse gives rise to a black hole in the space-time which covers the resulting space-time singularity. The ultimate fate of the star undergoing such a collapse is then an infinite curvature singularity at $r = 0$, which is completely hidden within the black hole. No emissions or light rays from the singularity could go out to observer at infinity and the singularity is causally disconnected from the outside space-time.

V. COSMIC CENSORSHIP

The question now is whether one could generalize these conclusions on the occurrence of a spacetime singularity in collapse and black hole formation for more general matter fields, and for non-spherical situations.

While we know that the occurrence of the singularity itself is stable to small perturbations in the initial data, there is no proof available that such a singularity will continue to be hidden within a black hole and remain causally disconnected from outside observers, even when the collapse is not spherical or when the matter does not have the form of exact homogeneous dust.

Thus, in order to generalize the notion of black holes to gravitational collapse situations other than exact spherically symmetric homogeneous dust case, it becomes necessary to rule out such naked or visible singularities by means of an explicit assumption. This is stated as the *cosmic censorship hypothesis*, which essentially states that if S is a partial Cauchy surface from which collapse commences, then there are no naked singularities to the future of S , that is, which could be seen from the future null infinity. This is true for the spherical homogeneous dust collapse, where the resulting spacetime is future asymptotically predictable and the censorship holds. Thus, the breakdown of physical theory at the spacetime singularity does not disturb prediction in future for the outside asymptotically flat region. What will be the corresponding scenario for other collapse situations, when inhomogeneities, non-sphericity etc are allowed for? It is clear that the assumption of censorship in a suitable form is crucial to basic results in black hole physics. In fact, when one considers the gravitational collapse in a generic situation, the very existence of black holes requires this hypothesis.

If one is to establish the censorship by means of a rigorous proof, that of course requires a much more precise formulation of the hypothesis. The statement that result of a complete gravitational collapse must always be a black hole and not a naked singularity, or all singularities of collapse must be hidden in black holes, causally disconnected from observers at infinity, is not rigorous enough. This is because, under completely general circumstances, the censorship or asymptotic predictability is false as one could always choose a space-time manifold with a naked singularity which would be a solution to Einstein's equations if we define $T_{ij} \equiv (1/8\pi)G_{ij}$. In fact, as far as the cosmic censorship hypothesis is concerned, it is a major problem in itself to find a satisfactory and mathematically rigorous formulation of what is physically desired to be achieved. Developing a suitable formulation would probably be a major advance towards the solution of

the main problem. It should be noted that presently no general proof is available for any suitably formulated version of the weak censorship.

VI. NAKED SINGULARITIES

This situation leads us to conclude that the first and foremost task is to carry out a detailed and careful examination of various gravitational collapse scenarios to examine them for their end states. It is clear from these considerations that we still do not have sufficient data and information available on the various possibilities for gravitationally collapsing configurations so as to decide one way or other on the issue of censorship hypothesis. What appears really necessary is a detailed investigation of different collapse scenarios, and to examine the possibilities arising, in order to have insights into the issue of the final fate of gravitational collapse. With such a purpose, several gravitational collapse scenarios involving different forms of matter have been investigated to understand better the final fate of collapse.

Since we are interested in collapse, we require that the space-time contains a regular initial spacelike hypersurface on which the matter fields, as represented by the stress-energy tensor T_{ij} , have a compact support and all physical quantities are well-behaved on this surface. Also, the matter should satisfy a suitable energy condition and the Einstein equations are satisfied. We say that the space-time contains a naked singularity if there is a future directed non-spacelike curve which reaches a far away observer or infinity in future, and in the past it terminates at the singularity.

As an immediate generalization of the Oppenheimer-Snyder-Datt homogeneous dust collapse, one could consider the collapse of inhomogeneous dust and examine the nature and structure of resulting singularity with special reference to censorship, and the occurrence of black holes and naked singularities. The main motivation to discuss this situation is this provides a clear picture in an explicit manner of what is possible in gravitational collapse. One could ask how are the conclusions given above for homogeneous collapse are modified when the inhomogeneities of matter distribution are taken into account. Clearly, it is important to include effects of inhomogeneities because typically a realistic collapse would start from a very inhomogeneous initial data with a centrally peaked density profile.

This problem was investigated in detail using the Tolman-Bondi-Lemaitre models, which describe gravitational collapse of an inhomogeneous spherically symmetric dust cloud (Joshi and

Dwivedi 1993). This is an infinite dimensional family of asymptotically flat solutions of Einstein equations, which is matched to the Schwarzschild spacetime outside the boundary of the collapsing star. The Oppenheimer-Snyder-Datt model is a special case of this class of solutions.

It is seen that the introduction of inhomogeneities leads to a rather different picture of gravitational collapse. The metric for spherically symmetric collapse of inhomogeneous dust, in comoving coordinates (t, r, θ, ϕ) , is given by,

$$ds^2 = -dt^2 + \frac{R'^2}{1+f} dr^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$T^{ij} = \epsilon \delta_t^i \delta_t^j, \quad \epsilon = \epsilon(t, r) = \frac{F'}{R^2 R'}$$

where T^{ij} is the stress-energy tensor, ϵ is the energy density, and R is a function of both t and r given by

$$\dot{R}^2 = \frac{F}{R} + f$$

Here the dot and prime denote partial derivatives with respect to the parameters t and r respectively. As we are considering collapse, we require $\dot{R}(t, r) < 0$. The quantities F and f are arbitrary functions of r and $4\pi R^2(t, r)$ is the proper area of the mass shells. The area of such a shell at $r = \text{const.}$ goes to zero when $R(t, r) = 0$. For gravitational collapse situation, we take ϵ to have compact support on an initial spacelike hypersurface and the space-time can be matched at some $r = \text{const.} = r_c$ to the exterior Schwarzschild field with total Schwarzschild mass $m(r_c) = M$ enclosed within the dust ball of coordinate radius of $r = r_c$. The apparent horizon in the interior dust ball lies at $R = F(r)$.

Using this framework, the nature of the singularity $R = 0$ can be examined. In particular, the problem of nakedness or otherwise of the singularity can be reduced to the existence of real, positive roots of an algebraic equation $V(X) = 0$, constructed out of the free functions F and f and their derivatives [11], which constitute the initial data of this problem. If the equation $V(X) = 0$ has a real positive root, the singularity could be naked. In order to be the end point of null geodesics at least one real positive value of X_0 should satisfy the above. Clearly, if no real positive root of the above is found, the singularity is not naked. It should be noted that many real positive roots of the above equation may exist which give the possible values of tangents to the singular null geodesics terminating at the singularity. Suppose now $X = X_0$ is a simple root to $V(X) = 0$. To determine whether X_0 is realized as a tangent along any outgoing singular

geodesics to give a naked singularity, one can integrate the equation of the radial null geodesics in the form $r = r(X)$ and it is seen that there is always at least one null geodesic terminating at the singularity $t = 0, r = 0$, with $X = X_0$. In addition there would be infinitely many integral curves as well, depending on the values of the parameters involved, that terminate at the singularity. It is thus seen that the existence of a positive real root of the equation $V(X) = 0$ is a necessary and sufficient condition for the singularity to be naked. Finally, to determine the curvature strength of the naked singularity at $t = 0, r = 0$, one may analyze the quantity $k^2 R_{ab} K^a K^b$ near the singularity. Standard analysis shows that the strong curvature condition is satisfied, in that the above quantity remains finite in the limit of approach to the singularity.

VII. GENERAL COLLAPSE SCENARIOS

The assumption of vanishing pressures, which could be important in the final stages of the collapse, may be considered as the limitation of dust models. On the other hand, it is also argued sometimes that in the final stages of collapse, the dust equation of state could be relevant and at higher and higher densities the matter may behave more like dust. Further, if there are no large negative pressures (as implied by the validity of the energy conditions), then the pressure also might contribute gravitationally in a positive manner to the effect of dust and may not alter the conclusions.

In any case, it is important to consider collapse situations with non-zero pressures and with reasonable equations of state. Pressures may play an important role for the later stages of collapse and one must investigate the possibility if pressure gradients could prevent the occurrence of naked singularity. These issues have been examined namely, the existence, the termination of non-spacelike geodesic families, and the strength of such a singularity for collapse with non-zero pressure. The results could be summarized as follows. If in a self-similar collapse with pressure, a single null radial geodesic escapes the singularity, then an entire family of non-spacelike geodesics would also escape provided the positivity of energy density is satisfied as above.

Actually, gravitational collapse models with a general form of matter, together with those such as directed radiation, dust, perfect fluids etc imply some general pattern emerging about the final outcome of gravitational collapse. Basically it follows that the occurrence of naked singularity is basically related to the choice of initial data to the Einstein field equations, and would therefore occur from regular initial data within the general context considered, subject to the matter satisfy-

ing weak energy condition. It appears that the occurrence of naked singularity or a black hole is more a problem of choice of the initial data for field equations rather than that of the form of matter or the equation of state. This has important implication for cosmic censorship in that in order to preserve the same one has to avoid all such regular initial data causing naked singularity, and hence a deeper understanding of the initial data space is required in order to determine such initial data and the kind of physical parameters they would specify. This would, in other words, classify the range of physical parameters to be avoided for a particular form of matter. More importantly, it would also pave the way for the black hole physics to use only those ranges of allowed parameter values which would produce black holes, thus putting black hole physics on a more firm footing.

What will be the final fate of gravitational collapse which is not spherically symmetric? The main phases of spherical collapse of a massive star would be typically instability, implosion of matter, and subsequent formation of an event horizon and a space-time singularity of infinite density and curvature with infinite gravitational tidal forces. This singularity may or may not be fully covered by the horizon as we have discussed above. Again, small perturbations over the spherically symmetric situation would leave the situation unchanged in the sense that an event horizon will continue to form in the advanced stages of the collapse.

The question then is, do horizons still form when the fluctuations from the spherical symmetry are high and the collapse is highly non-spherical? It was shown by Thorne (1972), that when there is no spherical symmetry, the collapse of infinite cylinders do give rise to naked singularities in general relativity, which are not covered by horizons. This situation motivated Thorne to propose the following *hoop conjecture* for finite systems in an asymptotically flat space-time, which characterizes the final fate of non-spherical collapse: The horizons of gravity form when and only when a mass M gets compacted in a region whose circumference in *every* direction obeys $C \leq 2\pi(2GM/c^2)$. Thus, unlike the cosmic censorship conjecture, the hoop conjecture does not rule out *all* the naked singularities but only makes a definite assertion on the occurrence of the event horizons in gravitational collapse. We also note that the hoop conjecture is concerned with the formation of event horizons, and not with naked singularities. Thus, even when event horizons form, say for example in the spherically symmetric case, it does not rule out the existence of naked singularities, i.e. it does not imply that such horizons must always cover the singularities.

Apart from such numerical simulations, some analytic treatments of aspherical collapse are also available. For example, the aspherical Szekeres models for irrotational dust without any Killing vectors, generalizing the spherical Tolman-Bondi-Lemaitre collapse, were studied, to deduce the

existence of strong curvature central naked singularities. While this indicates that naked singularities are not necessarily confined to spherical symmetry, it must be noted that dynamical evolution of a non-spherical collapse still remains a largely uncharted territory.

We note here that the *genericity* and *stability* of the collapse outcomes, in terms of black holes and naked singularities need to be understood carefully and in further detail. It is by and large well-accepted now, that the general theory of relativity does allow and gives rise to both black holes and naked singularities as final fate of a continual gravitational collapse, evolving from a regular initial data, and under reasonable physical conditions. What is not fully clear as yet is the distribution of these outcomes in the space of all allowed outcomes of collapse. The collapse models discussed above and considerations we gave here would be of some help in this direction, and may throw some light on the distribution of black holes and naked singularity solutions in the initial data space.

The important question then is the genericity and stability of such naked singularities arising from regular initial data. Will the initial data subspace, which gives rise to naked singularity as end state of collapse, have zero measure in a suitable sense? In that case, one would be able to reformulate more suitably the censorship hypothesis, based on a criterion that naked singularities could form in collapse but may not be generic.

It is natural to ask here, what is really the physics that causes a naked singularity to develop in collapse, rather than a black hole? We need to know how at all particles and energy are allowed to escape from extremely strong gravity fields. We have examined this issue in some detail to bring out the role of inhomogeneities and space-time shear to achieve this towards distorting the geometry of horizons forming in collapse. In Newtonian gravity, it is only the matter density that determines the gravitational field. In Einstein theory, however, density is just one attribute of the overall gravitational field, and the various curvature components and scalar quantities play an equally important role to dictate what the overall nature of the field is. What our results show is, once the density is inhomogeneous or higher at the center of collapsing star, this rather naturally delays the trapping of light and matter during collapse, which can in principle escape away. This is a general relativistic effect wherein even if the densities are very high, paths are created for light or matter to escape due to inhomogeneously collapsing matter fields, and these physical features naturally lead to a naked singularity formation rather than a black hole end state. It is the amount of inhomogeneity that counts to distort the horizons. If it is very small, below a critical limit, a black hole will form, but with sufficient inhomogeneity trapping is delayed to cause a naked singularity.

This criticality again comes out in the Vaidya class of radiation collapse models, where it is the rate of collapse, that is how fast or slow the cloud is collapsing, that determines the black hole or naked singularity formation.

VIII. DISTINGUISHING BLACK HOLES AND NAKED SINGULARITIES OBSERVATIONALLY

It is clear that the black hole and naked singularity outcomes of a complete gravitational collapse for a massive star are very different from each other physically, and would have quite different observational signatures. In the naked singularity case, if it occurs in nature, we have the possibility to observe the physical effects happening in the vicinity of the ultra dense regions that form in the very final stages of collapse. However, in a black hole scenario, such regions are necessarily hidden within the event horizon of gravity.

There have been attempts where researchers explored physical applications and implications of the naked singularities (see e.g. Joshi and Malafarina 2011 and references in there). If we could find astrophysical applications of the models that predict naked singularities as collapse final fate, and possibly try to test the same through observational methods and the signatures predicted, that could offer a very interesting avenue to get further insight into the problem as a whole. An attractive recent possibility in that connection is to explore the naked singularities as possible particle accelerators (Patil and Joshi 2011).

Also, the accretion discs around a naked singularity, wherein the matter particles are attracted towards or repulsed away from the singularities with great velocities could provide an excellent venue to test such effects and may lead to predictions of important observational signatures to distinguish the black holes and naked singularities in astrophysical phenomena. The question of what observational signatures would then emerge and distinguish the black holes from naked singularities is then necessary to be investigated, and we must explore what special astrophysical consequences the latter may have.

Where could the observational signatures of naked singularities lie? If we look for the sign of singularities such as the ones that appear at the end of collapse, we have to consider explosive and high energy events. In fact such models expose the ultra-high density region at the time of formation of the singularity while the outer shells are still falling towards the center. In such a case, shockwaves emanating from the superdense region at scales smaller than the Schwarzschild

radius (that could be due to quantum effects or repulsive classical effects) and collisions of particles near the Cauchy horizon could have effects on the outer layers. These would be considerably different from those appearing during the formation of a black hole. If, on the other hand, we consider singularities such as the superspinning Kerr solution we can look for different kinds of observational signatures. Among these the most prominent features deal with the way the singularity could affect incoming particles, either in the form of light bending, such as in gravitational lensing, particle collisions close to the singularity, or properties of accretion disks.

Essentially we ask whether we could test censorship using astronomical observations. With so many high technology power missions to observe the cosmos, can we not just observe the skies carefully to determine the validity or otherwise of the cosmic censorship? In this connection, several proposals to measure the mass and spin ratio for compact objects and for the galactic center have been made by different researchers. In particular, using pulsar observations it is suggested that gravitational waves and the spectra of X-rays binaries could test the rotation parameter for the center of our galaxy. Also, the shadow cast by the compact object can be used to test the same in stellar mass objects, or X-ray energy spectrum emitted by the accretion disk can be used. Using certain observable properties of gravitational lensing that depend upon rotation is also suggested (for references, see Joshi and Malafarina, 2011).

The basic issue here is that of sensitivity, namely how accurately and precisely can we measure and determine these parameters. A number of present and future astronomical missions could be of help. One of these is the Square-Kilometer Array (SKA) radio telescope, which will offer a possibility here, with a collecting area exceeding a factor of hundred compared to existing ones. The SKA astronomers point out they will have the sensitivity desired to measure the required quantities very precisely to determine the vital fundamental issues in gravitation physics such as the cosmic censorship, and to decide on its validity or otherwise. Other missions that could in principle provide a huge amount of observational data are those that are currently hunting for the gravitational waves. Gravitational wave astronomy has yet to claim its first detection of waves, nevertheless in the coming years it is very likely that the first observations will be made by the experiments such as LIGO and VIRGO that are currently still below the threshold for observation. Then gravitational wave astronomy will become an active field with possibly large amounts of data to be checked against theoretical predictions and it appears almost certain that this will have a strong impact on open theoretical issues such as the Cosmic Censorship problem.

There are three different kinds of observations that one could devise in order to distinguish a

naked singularity from a black hole. The first one relies on the study of accretion disks. The accretion properties of particles falling onto a naked singularity would be very different from those of black hole of the same mass (see for example (Pugliese et al, 2010; Joshi, Malafarina and Ramesh Narayan, 2011), and the resulting accretion disks would also be observationally different. The properties of accretion disks have been studied in terms of the radiant energy, flux and luminosity, in a Kerr-like geometry with a naked singularity, and the differences from a black hole accretion disk have been investigated. Also, the presence of a naked singularity gives rise to powerful repulsive forces that create an outflow of particles from the accretion disk on the equatorial plane. This outflow that is otherwise not present in the black hole case, could be in principle distinguished from the jets of particles that are thought to be ejected from black hole's polar region and which are due to strong electromagnetic fields. Also, when charged test particles are considered the accretion disk's properties for the naked singularity present in the Reissner-Nordstrom spacetime are seen to be observationally different from those of black holes.

The second way of distinguishing black holes from naked singularities relies on gravitational lensing. It is argued that when the spacetime does not possess a photon sphere, then the lensing features of light passing close to the singularity will be observationally different from those of a black hole. This method, however, does not appear to be very effective when a photon sphere is present in the spacetime. Assuming that a Kerr-like solution of Einstein equations with massless scalar field exists at the center of galaxies, its lensing properties are studied and it was found that there are effects due to the presence of both the rotation and scalar field that would affect the behavior of the bending angle of the light ray, thus making those objects observationally different from black holes.

Finally, a third way of distinguishing black holes from naked singularities comes from particle collisions and particle acceleration in the vicinity of the singularity. In fact, it is possible that the repulsive effects due to the singularity can deviate a class of infalling particles, making these outgoing eventually. These could then collide with some ingoing particle, and the energy of collision could be arbitrarily high, depending on the impact parameter of the outgoing particle with respect to the singularity. The net effect is thus creating a very high energy collision that resembles that of an immense particle accelerator and that would be impossible in the vicinity of a Kerr black hole.

IX. PREDICTABILITY AND OTHER COSMIC PUZZLES

What then is the status of naked singularities versus censorship today? Can cosmic censorship survive in some limited and specialized form, and firstly, can we properly formulate it after all these studies in recent years on gravitational collapse? While this continues to be a major cosmic puzzle, recent studies on formation of naked singularities as collapse end states for many realistic models have brought to forefront some of the most intriguing basic questions, both at classical and quantum level, which may have significant physical relevance. Some of these are: Can the super ultra-dense regions forming in a physically realistic collapse of a massive star be visible to far away observers in space-time? Are there any observable astrophysical consequences? What is the causal structure of space-time in the vicinity of singularity as decided by the internal dynamics of collapse which evolves from a regular initial data at an initial time? How early or late the horizons will actually develop in a physically realistic gravitational collapse, as determined by the astrophysical conditions within the star? When a naked singularity forms, is it possible to observe the quantum gravity effects taking place in the ultra-strong gravity regions? Can one possibly envisage a connection to observed ultra-high energy phenomena such as cosmic gamma ray bursts?

A continuing study of collapse phenomena within a general and physically realistic framework may be the only way to answers on some of these issues. This could lead us to novel physical insights and possibilities emerging out of the intricacies of gravitational force and nature of gravity, as emerging from examining the dynamical evolutions as allowed by Einstein equations.

Apart from its physical relevance, the collapse phenomena also have profound philosophical implications such as on the issue of predictability in the universe. We summarize below a few arguments, for and against it in the classical general relativity.

It is some times argued that breakdown of censorship means violation of predictability in space-time, because we have no direct handle to know what a naked singularity may radiate and emit unless we study the physics in such ultra-dense regions. One would not be able then to predict the universe in the future of a given epoch of time as would be the case, for example, in the case of the Schwarzschild black hole that develops in Oppenheimer-Snyder collapse.

A concern usually expressed is that if naked singularities occurred as the final fate of gravitational collapse, that would break the predictability in the spacetime, because the naked singularity is characterized by the existence of light rays and particles that emerge from the same. Typically,

in all the collapse models discussed above, there is a family of future directed non-spacelike curves that reach external observers, and when extended in the past these meet the singularity. The first light ray that comes out from the singularity marks the boundary of the region that can be predicted from a regular initial Cauchy surface in the spacetime, and that is called the Cauchy horizon for the spacetime. The causal structure of the spacetime would differ significantly in the two cases, when there is a Cauchy horizon and when there is none.

In general relativity, a given ‘epoch’ of time is sometimes represented by a spacelike surface, which is three-dimensional space. For example, in the standard Friedmann models of cosmology, there is such an epoch of simultaneity, from which the universe evolves in future, given the physical variables and initial data on this surface. The Einstein equations govern this evolution of universe, and there is thus a predictability which one would expect to hold in a classical theory. The concern that is expressed at times is one would not be able to predict in the future of naked singularity, and that unpredictable inputs may emerge from the same.

The point here is, given a regular initial data on a spacelike hypersurface, one would like to predict the future and past evolutions in the spacetime for all times (see for example, Hawking and Ellis 1973). Such a requirement is termed as the *global hyperbolicity* of the spacetime. A globally hyperbolic spacetime is a fully predictable universe, it admits a *Cauchy surface*, which is a three dimensional spacelike surface the data on which can be evolved for all times in the past as well as in future. Simple enough spacetimes such as the Minkowski or Schwarzschild are globally hyperbolic, but the Reissner-Nordstrom or Kerr geometries are not globally hyperbolic. For further details on these issues, we refer to (Joshi, 2008).

The key role that the event horizon of a black hole plays is that it hides the super-ultra-dense region formed in collapse from us. So the fact that we do not understand such regions has no effect on our ability to predict what happens in the universe at large. But if no such horizon exists, then the ultra-dense region might, in fact, play an important and even decisive role in the rest of the universe, and our ignorance of such regions would become of more than merely academic interest.

Yet such an unpredictability is common in general relativity, and not always directly related to censorship violation. Even black holes themselves need not fully respect predictability when they rotate or have some charge. For example, if we drop an electric charge into an uncharged black hole, the spacetime geometry radically changes and is no longer predictable from a regular initial epoch of time. A charged black hole admits a naked singularity which is visible to an observer within the horizon, and similar situation holds when the black hole is rotating. There is

an important debate in recent years, if one could over-charge or over-rotate a black hole so that the singularity visible to observers within the horizon becomes visible to external far away observers too.

Another point is, if such a black hole was big enough on a cosmological scale, the observer within the horizon could survive in principle for millions of years happily without actually falling into the singularity, and would thus be able to observe the naked singularity for a long time. Thus, only purest of pure black holes with no charge or rotation at all respect the full predictability, and all other physically realistic ones with charge or rotation actually do not. As such, there are many models of the universe in cosmology and relativity that are not totally predictable from a given spacelike hypersurface in the past. In these universes, the spacetime cannot be neatly separated into space and time foliation so as to allow initial data at a given moment of time to fully determine the future.

In our view, the real breakdown of predictability is the occurrence of spacetime singularity itself, which indicates the true limitation of the classical gravity theory. It does not matter really whether it is hidden within an event horizon or not. The real solution of the problem would then be the resolution of singularity itself, through either a quantum theory of gravity or in some way at the classical level itself.

Actually, the cosmic censorship way to predictability, that of ‘hiding the singularity within a black hole’, and then thinking that we restored the spacetime predictability may not be the real solution, or at best it may be only a partial solution to the key issue of predictability in spacetime universes. In fact, it may be just shifting the problem elsewhere, and some of the current major paradoxes faced by the black hole physics such as the information paradox, the various puzzles regarding the nature of the Hawking radiation, and other issues could as well be a manifestation of the same.

No doubt, the biggest argument in support of censorship would be that it would justify and validate the extensive formalism and laws of black hole physics and its astrophysical applications made so far. Censorship has been the foundation for the laws of black holes such as the area theorem and others, and their astrophysical applications. But these are not free of major paradoxes. Even if we accept that all massive stars would necessarily turn into black holes, this still creates some major physical paradoxes. Firstly, all the matter entering a black hole must of necessity collapse into a space-time singularity of infinite density and curvatures, where all known laws of physics break down, which is some kind of instability at the classical level itself. This was a reason

why many gravitation theorists of 1940s and 1950s objected to black hole formation, and Einstein also repeatedly argued against such a final fate of a collapsing star, writing a paper in 1939 to this effect. Also, as is well-known and has been widely discussed in the past few years, a black hole, by potentially destroying information, appears to contradict the basic principles of quantum theory. In that sense, the very formation of a black hole itself with a singularity within it appears to come laden with inherent problems. It is far from clear how one would resolve these basic troubles even if censorship were correct.

In view of such problems with the black hole paradigm, a possibility worth considering is the delay or avoidance of horizon formation as the star collapses under gravity. This happens when collapse to a naked singularity takes place, namely, where the horizon does not form early enough or is avoided. In such a case, if the star could radiate away most of its mass in the late stages of collapse, this may offer a way out of the black hole conundrum, while also resolving the singularity issue, because now there is no mass left to form the singularity.

What this means is, such an ‘unpredictability’ is somewhat common in general relativity. For example, if we drop a slight charge in a Schwarzschild black hole, the spacetime geometry completely changes into that of a charged black hole that is no longer predictable in the above sense. Similar situation holds when the black hole is rotating. In fact, there are very many models of universe in use in relativity which are not ‘globally hyperbolic’, that is, not totally predictable in the above sense where space and time are neatly separated so as to allow initial data to fully determine future for all times.

In any case, a positive and useful feature that has emerged from work on collapse models so far is, we already have now several important constraints for any possible formulation of censorship. It is seen that several versions of censorship proposed earlier would not hold, because explicit counter-examples are available now. Clearly, analyzing gravitational collapse plays a crucial role here. Only if we understand clearly why naked singularities do develop as collapse end states in many realistic models, there could emerge any pointer or lead to any practical and provable version of censorship.

Finally, it may be worth noting that even if the problem of singularity was resolved somehow, possibly by invoking quantum gravity which may smear the singularity, we still have to mathematically formulate and prove the black hole formation assuming an appropriate censorship principle, which is turning out to be most difficult task with no sign of resolve. As discussed, the detailed collapse calculations of recent years show that the final fate of a collapsing star could be a naked

singularity in violation to censorship. Finally, as is well-known and widely discussed by now, a black hole creates the information loss paradox, violating unitarity and making contradiction with basic principles of quantum theory. It is far from clear how one would resolve these basic troubles even if censorship were correct.

X. A LAB FOR QUANTUM GRAVITY–QUANTUM STARS?

It is believed that when we have a reasonable and complete quantum theory of gravity available, all spacetime singularities, whether naked or those hidden inside black holes, will be resolved away. As of now, it remains an open question if the quantum gravity will remove naked singularities. After all, the occurrence of spacetime singularities could be a purely classical phenomenon, and whether they are naked or covered should not be relevant, because quantum gravity will possibly remove them all any way. It is possible that in a suitable quantum gravity theory the singularities will be smeared out, though this has been not realized so far.

In any case, the important and real issue is, whether the extreme strong gravity regions formed due to gravitational collapse are visible to faraway observers or not. It is quite clear that the gravitational collapse would certainly proceed classically, at least till the quantum gravity starts governing and dominating the dynamical evolution at the scales of the order of the Planck length, *i.e.* till the extreme gravity configurations have been already developed due to collapse. The point is, it is the visibility or otherwise of such ultra-dense regions that is under discussion, whether they are classical or quantum (see Fig.2).

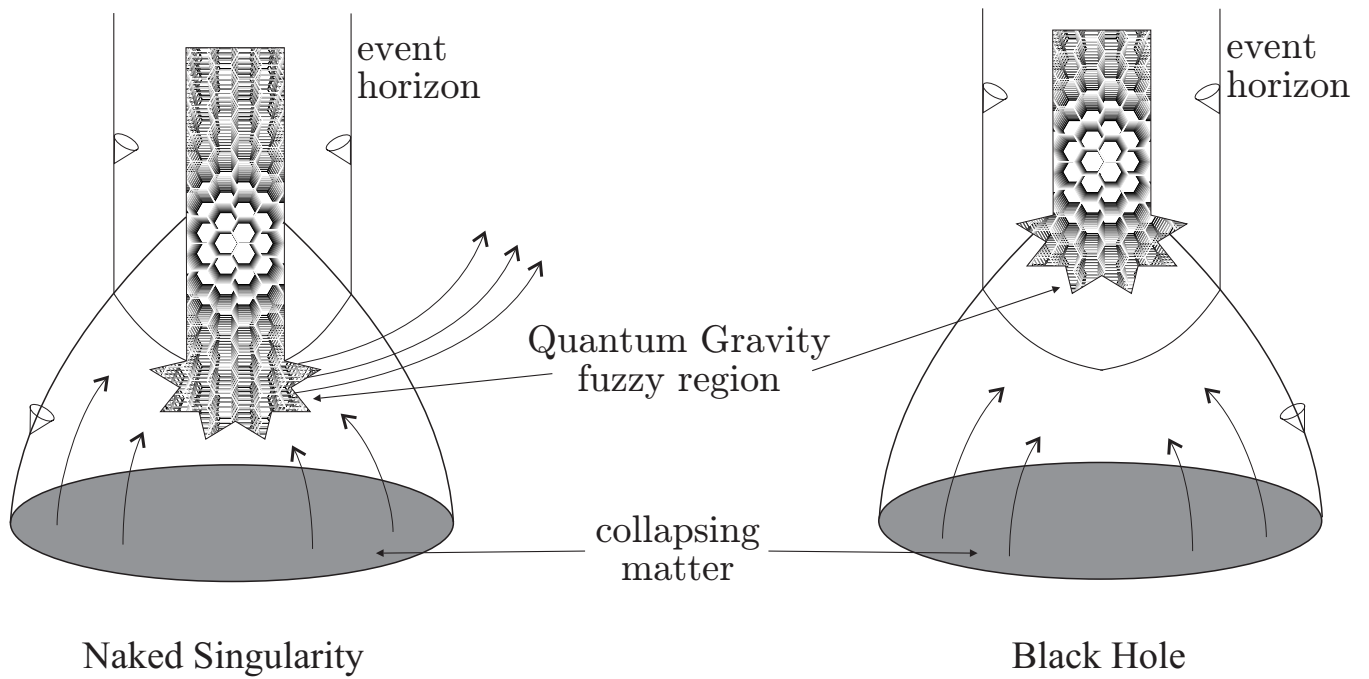


Fig 2: The naked singularity may be resolved by the quantum gravity effects but the ultra-strong gravity region that developed in gravitational collapse will still be visible to external observers in the universe.

What is important is, classical gravity implies necessarily the existence of ultra-strong gravity regions, where both classical and quantum gravity come into their own. In fact, if naked singularities do develop in gravitational collapse, then in a literal sense we come face-to-face with the laws of quantum gravity, whenever such an event occurs in the universe.

In this way, the gravitational collapse phenomenon has the potential to provide us with a possibility of actually testing the laws of quantum gravity. In the case of a black hole developing in the collapse of a finite sized object such as a massive star, such strong gravity regions are necessarily hidden behind an event horizon of gravity, and this would be well before the physical conditions became extreme near the spacetime singularity. In that case, the quantum effects, even if they caused qualitative changes closer to singularity, will be of no physical consequences as no causal communications are then allowed from such regions. On the other hand, if the causal structure were that of a naked singularity, then the communications from such a quantum gravity dominated extreme curvature ball would be visible in principle. This will be so either through direct physical processes near a strong curvature naked singularity, or via the secondary effects, such as the shocks produced in the surrounding medium. It is possible that a spacetime singularity basically represents the incompleteness of the classical theory and when quantum effects are combined with the gravitational force, the classical singularity may be resolved.

Therefore, more than the existence of a naked singularity, the important physical issue is whether the extreme gravity regions formed in the gravitational collapse of a massive star are visible to external observers in the universe. An affirmative answer here would mean that such a collapse provides a good laboratory to study quantum gravity effects in the cosmos, which may possibly generate clues for an as yet unknown theory of quantum gravity. Quantum gravity theories in the making, such as the string theory or loop quantum gravity in fact are badly in need of some kind of an observational input, without which it is nearly impossible to constrain the plethora of possibilities.

We could say quite realistically that a laboratory similar to that provided by the early universe is created in the collapse of a massive star. However, the big bang, which is also a naked singularity in that it is in principle visible to all observers, happened only once in the life of the universe and is therefore a unique event. But a naked singularity of gravitational collapse could offer an opportunity to explore and observe the quantum gravity effects every time a massive star in the universe ends its life.

The important questions one could ask are: If in realistic astrophysical situations the star terminates as a naked singularity, would there be any observable consequences which reflect the quantum gravity signatures in the ultra-strong gravity region? Do naked singularities have physical properties different from those of a black hole? Such questions underlie our study of gravitational collapse.

In view of recent results on gravitational collapse, and various problems with the black hole paradigm, a possibility worth considering is the delay or avoidance of horizon formation as the star evolves collapsing under gravity. This happens when collapse to a naked singularity takes place, where the horizon does not form early enough or is avoided. In such a case, in the late stages of collapse if the star could radiate away most of its mass, then this may offer a way out of the black hole conundrum, while also resolving the singularity issue, because now there is no mass left to form the curvature singularity. The purpose is to resolve the black hole paradoxes and avoid the singularity, either visible or within a black hole, which actually indicates the breakdown of physical theory. The current work on gravitational collapse suggests possibilities in this direction.

In this context, we considered a cloud that collapsed to a naked singularity final state, and introduced loop quantum gravity effects (Goswami, Joshi and Singh, 2006). It turned out that the quantum effects generated an extremely powerful repulsive force within the cloud. Classically the cloud would have terminated into a naked singularity, but quantum effects caused a burstlike

emission of matter in the very last phases of collapse, thus dispersing the star and dissolving the naked singularity. The density remained finite and the spacetime singularity was eventually avoided. One could expect this to be a fundamental feature of other quantum gravity theories as well, but more work would be required to confirm such a conjecture.

For a realistic star, its final catastrophic collapse takes place in matter of seconds. A star that lived millions of years thus collapses in only tens of seconds. In the very last fraction of a microsecond, almost a quarter of its total mass must be emitted due to quantum effects, and therefore this would appear like a massive, abrupt burst to an external observer far away. Typically, such a burst will also carry with it specific signatures of quantum effects taking place in such ultra-dense regions. In our case, these included a sudden dip in the intensity of emission just before the final burstlike evaporation due to quantum gravity.

The question is, whether such unique astrophysical signatures can be detected by modern experiments, and if so, what they tell on quantum gravity, and if there are any new insights into other aspects of cosmology and fundamental theories such as string theory.

The key point is, because the very final ultra-dense regions of the star are no longer hidden within a horizon as in the black hole case, the exciting possibility of observing these quantum effects arises now, independently of the quantum gravity theory used. An astrophysical connection to extreme high energy phenomena in the universe, such as the gamma-rays bursts that defy any explanations so far, may not be ruled out.

Such a resolution of naked singularity through quantum gravity could be a solution to some of the paradoxes mentioned above. Then, whenever a massive star undergoes a gravitational collapse, this might create a laboratory for quantum gravity in the form of a *Quantum Star* (see e.g. Joshi, 2009), that we may be able to possibly access. This would also suggest intriguing connections to high energy astrophysical phenomena. The present situation poses one of the most interesting challenges which have emerged through the recent work on gravitational collapse.

We hope the considerations here have shown that gravitational collapse, which essentially is the investigation of dynamical evolutions of matter fields under the force of gravity in the spacetime, provides one of the most exciting research frontiers in gravitation physics and high energy astrophysics. In our view, there is a scope therefore for both theoretical as well as numerical investigations in these frontier areas, which may have much to tell for our quest on basic issues in quantum gravity, fundamental physics and gravity theories, and towards the expanding frontiers of modern high energy astrophysical observations.

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