Eighth Vaidya-Raychauduri Endowment Award Lecture

Gravity and Light

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Indian Association for General Relativity and Gravitation

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In the beginning

It is both a privilege and a challenge to deliver the Vaidya– Raychaudhuri endowment lecture before this gathering of the Indian Association for General Relativity and Gravitation. The overlap with relativity in my work has been via the areas of accretion onto black holes and gravitational lensing, and to some extent cosmology. That only makes me a consumer rather than a producer of relativity. I have however spent a fair time on the other half of the announced theme — namely light. The temptation to explore the connection with gravity stems from my own curiosity about that most beautiful of physical theories, Einstein's General Theory of Relativity (GTR). Napoleon remarked that every soldier of France carries the marshal's baton in his knapsack. The equivalent for most physicists is, in youth, to buy and dip into a GTR book which you can see on their shelves long after they have given it up. In my days and earlier, it used to be "The Meaning of Relativity" by Albert Einstein, or Eddington's MTR — Mathematical Theory of Relativity. A decade or so later, it was more likely to be the redoubtable MTW — Misner Thorne Wheeler — which I have seen testing the strength of the bookshelf in the office of one of my friends who is a well known chemical physicist. (Not that relativists or astrophysicists can afford to be condescending to chemists after the debt that relativistic astrophysics owes to a physical chemist by the name of Zeldovich). Another friend of mine now in condensed matter actually started in relativity before, as he put it, he "lapsed and shifted". I confess that I share this widespread fascination for

GTR. I was fortunate to spend a few decades at the Raman Research Institute where it was possible to learn something of GTR by osmosis, since there were usually always three or four practicing, card holding relativists available for discussions.

Coming now to the theme I have chosen, "Gravity and Light". we know that Newton thought about both gravity and light and expressed his own conviction that they were connected in the the very first of his famous thirty one Queries (which I recommend to all of you) at the end of his book "Opticks" [1].

"Do not bodies act upon light at a distance, and by their action bend its Rays; and is not this action (caeteris paribus) strongest at the least distance?".

Clearly, he would not have been surprised by gravitational lenses. Some of Newton's later queries can be interpreted as speculating on the second law of thermodynamics, mass–energy equivalence, and the cosmological constant. Clearly, he was not sticking narrowly to the title of the book! His Queries are not meant to be speculations offered in a spirit of humility, but are rhetorical questions, outlining a theory of everything which he knew he would not complete in his lifetime.

Newton's lead was followed up a hundred years later [2] by the Reverend John Michell (1784) and the revered Pierre–Simon Laplace (1796). Both speculated on bodies which were so concentrated that light could not escape from them — in modern language, black holes. Their picture is not the same as the modern one, because they used the escape velocity concept and had the light going out and falling back, but they did have the correct formula for the Schwarzschild radius. The so called "Newtonian" formula for the deflection of light was also calculated well before Einstein. It only required one to be naive enough to extrapolate the deflection of a nonrelativistic particle of speed v to $v = c$. Like Michell and Laplace, one had to ignore the fact that in such a Newtonian picture, light would not travel at the speed of light. Today, we should teach these as dimensional arguments at best, which tell us that there is an important length $R = 2GM/c^2$ associated with a body of mass M. Purely on dimensional grounds, the formula for the deflection of a particle of speed v could always have some power of $(1-v^2/c^2)$ in it and we could end up with zero deflection for light. This is exactly what happens if we construct a scalar theory or even a vector theory (modeled on Maxwell's) of gravitation, based on special relativity [3]. Such theories can only be ruled out by experiment, real or gedanken. What our teaching should emphasise is that the deflection of light is very specific to a tensor theory, which in turn is closely tied to the equivalence principle. This kind of argument is given in the Zeldovich–Novikov book [4], and people with a special relativity based particle physics training find them natural. Recall that the two volumes on Relativistic Astrophysics were written towards the end of Zeldovich's particle physics phase. What is remarkable from this viewpoint is that in such a theory, the role of special relativity changes completely — it is still there, but only in the small, not in the large. One can see this point being faced, dare I say with some discomfort, even in Weinberg's book [5] on gravitation and cosmology. Of course, the particle physicists view of gravity has gone a long way since that book appeared. First gauge theory and then string theory have geometrised particle theorists to an extent that their forebears of the sixties would not recognise them. Given half a chance, they would cheerfully wrap up spacetimes of any dimension into any shape.

Einstein's derivation of the "Newtonian" deflection formula in 1908 was not as naive as the earlier ones. It was based on the equivalence principle — or in popular language his elevators. The fact that he got half the deflection should not be held against him — space curvature had not yet appeared in 1908. But we must agree that fortune favoured him. An eclipse visible from Russia in 1912 fortunately eluded study because of the disturbed political conditions there. This meant that Einstein could have the full GTR formula in place, $\theta = 4GM/c^2b$ before the next successful eclipse expedition of 1919. It is interesting to speculate what would have happened in 1912 if Einstein had to contend with a measured deflection double of what his current theory predicted. I suspect it would not have made much difference, because he would have been uncomfortable with his incomplete theory anyway. As a matter of fact, one of the two sub-expeditions of 1919, taken by itself, tended to favour the "Newtonian" deflection formula! [6]. But with Eddington as the head of the other team, the conclusion was foregone. Has it not been said that one should never believe an experiment until it agrees with theory?

Black holes as wavefronts

With the gravitational deflection formula in place, we now go back to the black hole, i.e to the Schwarzschild solution of 1915. Here, the remarkable fact is, (as stated tersely by Landau and Lifshitz [7]), that the physical meaning of the Schwarzschild solution was first given by D.Finkelstein in 1958 [8]. Why did it take forty three years? Because of the confusion caused by the freedom to choose different co-ordinate systems. Finkelstein needed co-ordinates attached to ingoing light rays. Eddington and Lemaitre who had such null co-ordinates before Finkelstein did not go all the way to the black hole. Let us also remember that Vaidya's metric, given in the nineteen forties, already used outgoing null co-ordinates! And the full view — should we call it Vishwarup? — of the Schwarzschild solution, needs both kinds of co-ordinates (read Box 31.1, Reference [3] for the history). So the tradition of exploring spacetime with light rays, set by Einstein in his special theory, continued to enrich GTR as well.

I was fortunate to hear many of these things from Vishveshwara and his younger colleagues at RRI in the early nineteen eighties. His Einstein Centenary lecture on Black holes for Bedtime [9] should be required reading for serious students, who should not be misled by its delightfully light touch. Of course, the lecture should be followed up with some library work and calculation. Even after doing all this one needs time to get used to the idea that the horizon of the Schwarzschild black hole is an outgoing, propagating spherical wavefront of light which is just unable to increase its radius. Vishveshwara's lecture evokes Lewis Carroll's Alice who says that one has to move as fast as one can to stay in the same place. Sometimes, I try to imagine someone trying to run up a descending escalator. Unruh drew a parallel, many years ago, with the sonic point of an infalling, accelerating fluid, which marks a kind of horizon, since sound waves cannot carry information from inside to outside. In the blackhole case, disconcertingly, the fluid is vacuum. And inside this surface, the spacetime is not stationary. One had to keep rubbing ones eyes and reminding oneself that inside the horizon, the particle moving to smaller r is actually moving towards the future, rather than towards the centre! Such is the power of symbols over our minds that perhaps the interior should be written with r and t interchanged. All this should seem trivial to today's students of GTR who learn that the spacetime manifold comes first and the co-ordinates come later, sometimes only in shreds and patches, and they might well feel that I am labouring the point. But you should not think that enlightenment spread instantly after Finkelstein's paper. I have attended at least one lecture in the early seventies which spoke of the Schwarzschild singularity at $r = 2GM/c^2$. And one claim was made in the not so early eighties for an equilibrium configuration inside the Schwarzschild radius, which missed the fact that the spacetime was not globally stationary.

Interestingly, astronomers seem to have embraced the black hole well before the physicists. The 2002 Physics Nobel prize reminds us of the early discoveries of X-ray astronomy, which included some of the best black hole candidates in the stellar mass range. The evidence for the black hole was partly an absence of competing models. Similarly, in the case of quasars and radio galaxies, the black hole was used to build models whose consistency could be broadly checked, but the "no credible alternative" syndrome also played a role. Today, the case for black holes on a galactic scale has become much stronger. Beautiful observations have shown gas and stars in orbit around the centers of other galaxies and our own, with velocities of thousands of kilometres per second, far faster than any other viable non black hole model of the central region could account for [10].

I would now like to turn to gravitational lensing, which is the inaccurate but prevalent name for the whole range of phenomena in astronomy associated with the gravitational deflection of light [11]. In the broadest sense, the original eclipse test also comes into this category. One usually thinks of the deflection of light grazing the edge of the sun by 1.75 arc seconds as a small effect. It was therefore a good lesson to talk to a radio astronomer who does VLBI — very long baseline interferometry — where angles smaller than a milliarcsecond are routinely measured. I told him I worked on gravitational lensing and he nodded understandingly — "It's such a huge effect, isn't it?". Indeed, a ray reaching the earth 90 degrees away from the Sun suffers a deflection of five milliarcseconds, one four hundredth of the value for a ray grazing the Sun. So here is one community which puts the gravitational deflection of light into all their routine calculations without a second thought. They could get an error of more than 2π radians of phase if they neglected it.

The other reason why gravitational deflection can have "huge" consequences is simply distance. To appreciate this, take what you think is a good mirror, and walk away from it. The small deviations from a plane hardly matter at shaving distance, but are usually quite spectacular at five metres, at least with the mirrors that I tend to buy. Einstein realized that rays from a star passing the edge of the Sun would cross at an observer located somewhat more than 100,000 astronomical units away. To such an observer, rays from a single favourably placed object would form a ring, of angular radius 1.75 arc seconds. This is the famous Einstein ring, which is the simplest, most symmetric case of gravitational lensing. Notice that other rays do not focus at the same point, so the "lens" suffers from severe spherical aberration. Einstein himself seems to have regarded it as a curiosity. Russell (of the Hertzsprung–Russell diagram fame) described the phenomenon in some more detail for the case of one star travelling behind another. His choice of journal (Scientific American) and subtitle (some impossible tests of general relativity) showed that he shared Einstein's view. How would both Einstein and Russell react to see dozens of papers on the observation and theory of this phenomenon (called microlensing) appear every year? It is not a test but an application. There is an old adage of experimental particle physics that yesterday's discovery is today's calibration and tomorrow's background, and this is happening to lensing. For those who remember the excitement which accompanied the discovery of each new gravitational lens in the nineteen eighties, it is interesting that just a month ago the results of a whole survey of lenses was announced. The statistics — the number of lenses found and their angular sizes, were taken as evidence of a significant role for the cosmological constant in the current epoch [12]

The original evidence for the cosmological constant came a few years ago, from the apparent brightness of Type Ia supernovae as a function of redshift [13]. I would like to argue that this too is a form of gravitational lensing. The energy we receive per unit area is inversely proportional to the area of the sphere over which the energy from the supernova has been spread, in its journey to us. The area of this sphere is related to how much distance two rays at a given angle to each other diverge. And this divergence is related to the mass–energy in the beam, as the Raychaudhuri equations tell us [14]. This is not gravitational lensing by a discrete object, but it is gravitational lensing by the whole universe. Incidentally, the textbook derivation of angular size distance and luminosity distance assumes that the we have a universe which is uniform even on the scale of the two rays of light which are diverging from each other. One should worry about the correctness of this assumption when we are looking at a relatively small angular size object like a distant quasar — can we really use this smoothed out picture? This effect was pointed out independently by Feynman and Zeldovich in the early nineteen sixties [15]. Characteristically, Feynman's remark was unpublished by him, but was acknowledged in the PhD thesis and corresponding papers of a young theorist at Caltech, James Gunn Today, this "theorist" is at in the vanguard of the most ambitious observational optical survey ever made, the Sloan [16], which has detected the kind of effects anticipated by him and others forty years earlier. With current models, the cosmological constant makes a smooth contribution to the divergence of light rays, but the matter is clumped on different scales, so these effects could still be important in precise work, which is more and more the case in cosmology.

Relativistic Optics

Most astrophysical calculations of gravitational lensing are based on a weak field solution for the light bending near the object, and light travelling in a standard cosmological solution far away from the object. One can therefore work on lenses without worrying about niceties of how light should be described in GTR. I guess my curiosity about GTR and the chance of learning it straight from the mouths of several horses at RRI exposed me to some of these niceties, which are worth dwelling on in a lecture on gravity and light. Some of the points made are relevant even to the discussion of light in special relativity but SR is after all, the material of which gravity is woven in GR. One of the first things I was told on the top floor of the RRI Library Block — and it wasn't obvious at all — was that distances perpendicular to the direction of propagation, or more graphically, shadows, were Lorentz invariant. One can thus talk of the area of a piece of wavefront without any offence to the principles of relativity. I remember Bala Iyer giving me a well thumbed copy of an article by Frolov in the Trudy (proceedings) of the Lebedev institute which gave the necessary theory.

Equally shocking to a beginner was the news that we are not allowed to talk about the distance which a light ray has travelled — any attempt to construct a Lorentz invariant formula for this distance results in a zero answer, because light travels along a null vector. There is a substitute for distance, called affine length. It would not do to define this to an audience of relativists. In fact, it would be more appropriate to define relativists as people who are completely comfortable with affine length. One more shock regarding light in special relativity was administered by Penrose in 1956. If one concentrates just on the direction of arrival of rays of light coming from the backward light cone, these form a sphere. In fact, this is what astronomers call the celestial sphere. When one carries out a Lorenz boost, this sphere is mapped onto itself by aberration — surely one of the oldest effects in our subject!

Fully relativistic aberration has a remarkable, exact property — it is a conformal transformation on the unit sphere, or on the complex plane to which it can be mapped by Riemann's stereographic projection. A cone of rays, forming a small (not infinitesimal, but in contrast to great) circle on this sphere, maps into another cone, something which is easy to check once one is told that it is so. This means that if you look at a moving ball of any size, its outline is still a circle, contrary to what popular books including those by the legendary George Gamow had said up to that time. We know that Einstein was stunned by Thomas precession, a direct consequence of special relativity, even though he said in his first paper that the Lorentz transformations form a group. As the date 1956 tells you, we will never know what his reaction would have been to the Penrose theorem on the visual appearance of a moving sphere.

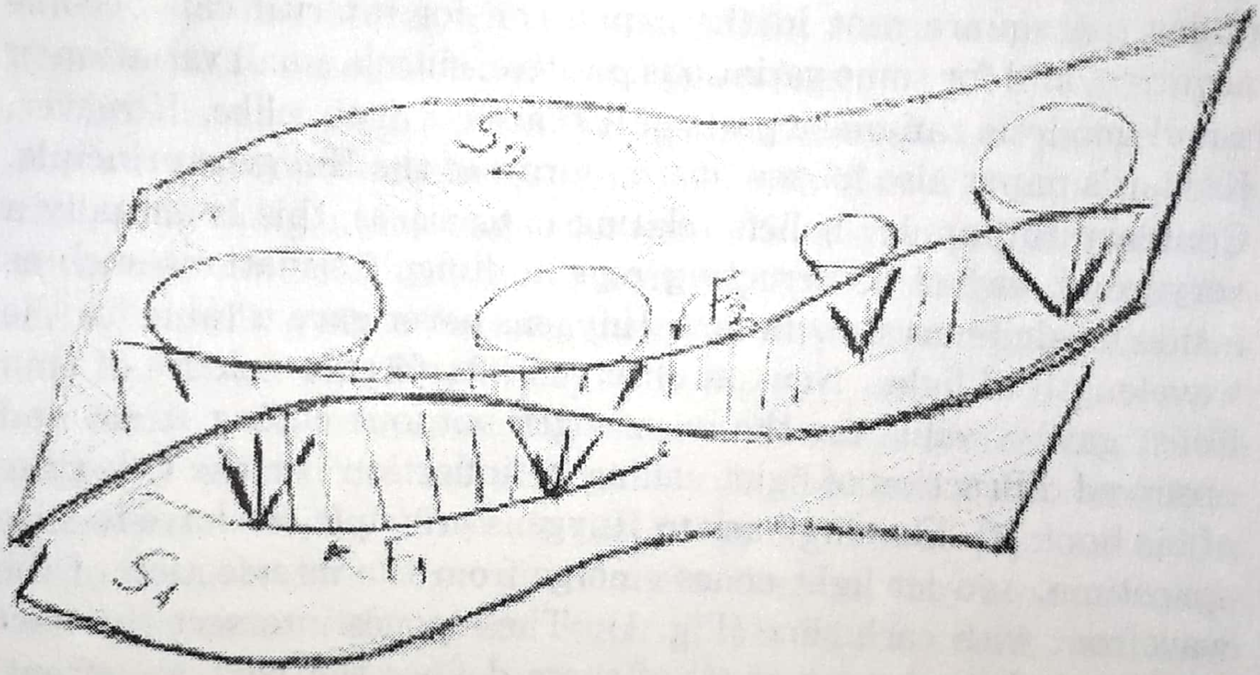

Figure 1: Huygen's principle in general relativity F_1 is a wavefront on a slice S_1. Null cones from all points of F_1 cut a neighbouring slice S_2, and the envelope of these intersections in the new wavefront F_2. F_1F_2 etc., sweep out a null surface, ruled by null geodesics (light rays) which at tangent to the local light cone. The surface itself is independent of slicing.

Another shock to an optician learning relativity is that one has to give up the notion of adjacent rays. On the light cone, either two rays are identical, or they are different. Given three rays, it makes no sense to say that A is close to B but far away from C. There is always a frame of reference in which A and B travel in opposite directions, and hence appear as far apart as two rays of light could possibly be. Joseph Samuel and I encountered some of these issues in our exploration [20] of Fermat's principle in general relativity, and I would like to spend some time on that. As I mentioned earlier, I and many others in the field of lensing were quite happy to work in a specific co-ordinate system and treat the angles, wavefronts, etc. in the way one normally does in optics. One of my fellow lensmen, Israel Kovner (from Israel, I should add) tried to do Fermat's principle covariantly, but ran into the technical problem of varying the invariant distance which was zero for a light ray. For some variations of a null curve, the quantity under the square root in the expression for interval can become negative, and for some variations positive, since a small variation of a null geodesic can make parts of it spacelike or timelike. However, Kovner's paper also looked at it in terms of the Huygens principle. Contrary to popular beliefs relating it to *waves*, this is actually a very good way of constructing *rays* in difficult situations such as inside a calcite crystal. In fact, Huygens never gave a value for the wavelength of light. Newton observed interference colours of thin films, gave a value for the wavelength without calling it so, and observed diffraction of light, calling it "inflection" on the title page of his book [1]. Coming back to Huygens principle, we have to slice spacetime, and let light cones emerge from the intersection of the wavefront with each slice (Fig. 1). These cones intersect the next slice, and the envelope of all of these defines the next wavefront. Notice that we should be sure our answer doesn't depend on the slicing. One way to see, if not prove, the slice independence is to think of the wavefront as enclosing the set of all events which can be causally influenced from the first wavefront. Now where does Fermat come in? Each point on a given wavefront has a corresponding point on the next wavefront. Joining them up defines the rays. In four dimensions, we have no right to say that the ray is the shortest distance between two wave fronts. What we can say is that light arrives first along the actual ray, as compared to any other null curve. On this three dimensional object called the wavefront, the rays are "null generators", i.e. the ray through any point is a null direction through that point lying on the surface.

The equation which goes with these pictures was given by Hamilton nearly two hundred years ago, and is known as the eikonal equation for the phase S as a function of space and time. (The generalisation to particle mechanics goes by the name of the Hamilton–Jacobi equation). Since we are being fully four dimensional here, there is no question of a time independent equation. The function S, being the phase of the wave, is a Lorentz invariant, which is constant on three dimensional wavefronts. The eikonal equation simply tells us that the four dimensional gradient of S is a null vector. Being a covariant vector, it doesn't actually point anywhere, but can be raised to contravariant status with the help of the metric to point along the null generator.

So far, we have looked at things in the small, i.e. just evolved the wavefront between neighbouring slices. But if we go on doing this, something interesting can happen. In a specific frame of reference, this can be illustrated by drawing rays normal to wavefronts. After travelling some distance they start trying to cross each other and form caustics (Fig. 2) [18].

What is the corresponding statement in four dimensions? Two adjacent null generators disappear into the light cone, and then more and more do. If we are just mapping causal domains in the spacetime, then we forget about them, because they are now inside the light cone, visiting spacetime points which could have been reached by mere timelike mortals. But if you are interested in astronomy , then you don't forget them, because they are delayed and sometimes inverted images, which have the added advantage of being rather bright and easy to detect. The first astronomer to take this possibility seriously was Zwicky in 1937, and his dream of using gravitational lenses as telescopes is being realised today. Clusters of galaxies are good candidates, which have focal lengths less than the Hubble scale (both in terms of affine parameter, if

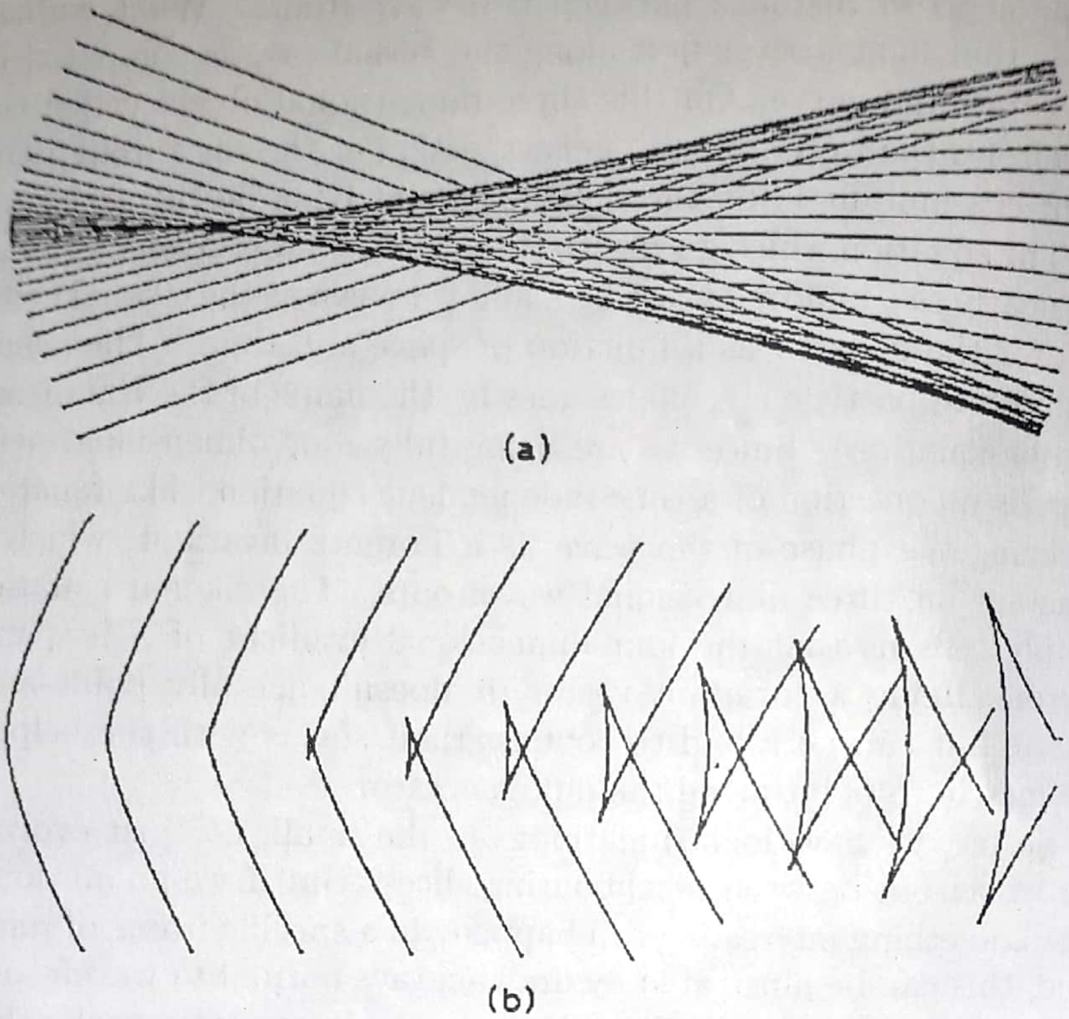

Figure 2: The relationship between rays, and caustics (a) and wavefronts (b) illustrated for two space dimensions. Notice several general features (i) Multiple intersecting rays i.e., multiple images for observers within the caustic (ii) Merging of a pair of images as the observer approaches the caustic (and merging of three at the cusp) (iii) Crowding of rays corresponding to high intensities at the caustic.(iv) Part of the wavefront enclosed inside the rest, corresponding to rays which have come inside the outer causal boundary. These will be seen as images delayed with respect to the first one to arrive.

you insist). The criterion for focusing is an interesting one. If one neglects what opticians call astigmatism and relativists call shear, then you need a critical mass density per unit area to get a focal length less than the Hubble scale, and this comes out to be proportional to cH_0/G, and numerically not far from one gram per square centimetre, a great relief from most astronomical numbers.

The Importance of Shear

As with light deflection, dimensional analysis does not tell the whole story. The loophole is shear — that one can focus in one direction and defocus in the other. The net effect is still to focus — after all, if we start with a beam of square cross section, and one side doubles while the other shrinks to zero, the area has become zero as well, and the intensity infinite. The sum of the two wavefront curvatures can be enhanced by surface density, which is what we normally call focusing of light by the gravity of matter. But even if there is zero surface density in the beam, we could have compression in one direction and expansion in the other. This is sometimes called Weyl focusing, and one can think of it as shear of the shape of the beam in the transverse plane, caused by tidal forces coming from matter outside the beam. This is a non local effect, and it turns out that even a very small surface density can produce a large shear, though somewhat artificially from an astronomical point of view. As with many things about lensing, getting things straight from the horses mouth helped (the horses in this case being Subramanian and Padmanabhan, see [19])

In order to see whether a beam has been sheared, one has to know what its shape was to start with, a luxury that we do not have in any individual case. But nearly twenty years ago, Tyson and colleagues at Bell labs decided to look at the shapes of thousands of galaxy images. The assumption was that their average should be circular if there was no shear in the propagation from source to us. Clearly, one needs large numbers to see any systematic effect over and above the random one coming from the fact that galaxy images are not circular but rather elliptical, with (hopefully!) random orientations. The first successful reports came from three independent groups about the year 2000. Not only was the systematic shear detected, but the statistical properties of its spatial variation could be studied [20]. These statistical properties are predicted to be different in different cosmological models, and seem to favour dark matter with cosmological constant, already mentioned in the context of the apparent luminosity of supernovae, and gravitational lensing by galaxies.

On the scale of clusters of galaxies, we have a rare situation — one is able to compare estimates of mass and its distribution from three different kinds of observations — hot gas which is more or less in hydrostatic equilibrium, the dynamics of galaxies, and the shear and convergence of background galaxy images produced on passing photons. The measurement of convergence requires some explanation. If a circular beam undergoes convergence/divergence, it remains circular. How then does one know how much convergence or divergence has occured? There are two ways. One is that there is a global (i.e. not local) relation between shear on the one hand, and convergence on the other. Roughly speaking, in the two dimensional plane of the wavefront, convergence (a scalar quantity) is like the "charge" or "source" and shear (a vector like quantity) is like the electric field. This astute observation by Kaiser and Squires [21] in 1993 has been used ever since, with increasing success, in reconstructing the mass distribution of clusters of galaxies. This is derived in the Newtonian, weak field framework, and I am not aware of a fully relativistic analogue. The other independent test is that if a region of the sky has undergone convergence, the mean number and brightness of galaxies seen through that region will be different from the average. The technique of studying the mass distribution in the universe by means of the statistics of a huge number of images is bound to grow and flourish as still better detectors and computers come on the scene. It is not so different from the old fashioned way of telling fortunes by the way tea leaves lie at the bottom of the pot!

A brief look at polarisation

We now come to the polarisation of light. One might think that since Huygens solved the problem of the double refraction of calcite by his construction, that he would have thought of polarisation, but the fact is that he missed it completely. Newton was quantitatively wrong on the law for double refraction, but listen to this extract from his Query number twenty six [1].

"Every Ray of Light has therefore two opposite Sides, originally endowed with a Property on which the unusual Refraction depends, and the other two opposite sides not endowed with that property"

Notice that his description of polarisation as light having "two opposite sides", is more like a second rank tensor than a vector, which is the correct way of describing the statistical properties of polarised light. Notice also that this has the same transformation properties with respect to rotations around the direction of propagation as the shear of a beam. In particular, the two principal directions (in which the intensity transmitted by a polaroid are minimum and maximum) move along the beam by parallel transport. This is actually useful in interpreting gravitational lensing observations, where it is useful to have one more check of the identical origin of two images. Two other tests are of course spectrum and surface brightness. The latter, being intensity per unit solid angle requires that we resolve the image, which is more common in radio observations of extended lensed objects.

Since Fermat's principle allows us to consider null curves which are not geodesics, it seemed natural to try and transport polarisation along these null curves. In the case of timelike non-geodesics, the spin vector evolves by Fermi Transport [5]. One more of the rude shocks of relativistic optics is that this breaks down for null curves. Samuel wrote down - literally - a transport law along null curves, which he called "Kattabomman Transport" after the legendary freedom fighter whose name used to grace buses in Kanyakumari district of Tamil Nadu till a few years ago. We managed to see that this emerged from the most natural rule for transporting vectors (or spinors, which is more general) from one null ray to any other (not adjacent!) null ray [22]. We are convinced that it is geometric and natural, and have only one problem — to find a killer application.

Conclusion and acknowledgements

Today, we consider it natural that students of astronomy learn about Kepler orbits since that is the basic way that masses of stars are determined. The time has come for every student of astronomy who wants to know how mass is to be measured on scales larger than an individual galaxy (and sometimes smaller as well!) to undergo a basic course entitled (what else?) "Gravity and light".

Both gravity and light need sources. I have done a fair job of acknowledging my sources for the gravity part, including lensing, (and have done better in [18]). Coming to light, I must have learnt everything I know in the classrooms of Profs. Lakshminarayanan and Srinivasan (KL and RS to their students at Vivekananda College and IIT Madras), and in the research environment at NAL and RRI from Profs. Ramaseshan, Ranganath, and Radhakrishnan.

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