Second Vaidya-Raychaudhuri Endowment Award Lecture

Gravity as a Physical Interaction

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Indian Association for General Relativity and Gravitation

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Gravity as a Physical Interaction

The honour at being asked to deliver the Second Vaidya – Raychaudhuri Lecture naturally brings with it the feeling of responsibility of saying something wise for the occasion that is befitting the stature of these two doyens of Indian Relativity. For me it also brings a feeling of inadequacy especially since I have to do the impossible task of matching the high standard set by Professor Mukunda at the First V-R Lecture. Nevertheless I will do my best : and I will enjoy the experience since it gives me the opportunity of doing some loud thinking. Both Professors Vaidya and Raychaudhuri are reputed teachers besides being renowned researchers. As such they have that admirable but scarce quality of patience which teachers need to listen to immature or half–baked ideas of their pupils. I hope they will give me a patient hearing.

Non fingo hypotheses

This is what Isaac Newton is reputed to have said when asked to give his views on why the law of gravitation follows the inverse square law that he had proposed. By refusing to hypothesise about gravity Newton was opting for the realist/pragmatic approach, an approach that seeks and stops at finding the underlying rule that best describes a variety of natural phenomena. In a letter to Richard Bentley dated the 17th January, 1692 (93) he wrote

> *"You sometimes speak of gravity as essential and inherent to matter : pray do not ascribe that notion to me, for ye cause of gravity is what I do not pretend to know and therefore would take more time to consider of it."*

Although, it is interesting to note that fourteen years earlier in a letter to Robert Boyle [dated February 28, 1678 (79)] Newton had, somewhat hesitantly, offered an etherbased theory of gravity. It is important to recognize that Newton's law of gravitation marks the next step after Kepler's laws which in turn were one step ahead of the Copernican revolution. Copernicus rightly identified the Sun as the centerpoint of the solar system. However, he still retained the epicycles, the 'circles upon circles' so popular with the Greeks. Thus his planetary trajectories were quite combersome and still not exactly right. Kepler's main achievement was in breaking away from the Aristotelian preferences for circles as trajectories for natural motion. By stating that planets move in elliptical orbits with the Sun as one of the two foci, Kepler made it easy for Newton to *deduce* the inverse square law as the rule governing the dependence of the force of gravity on the distances between the Sun and the planets.

Today a student who knows ordinary differential equations and who knows the laws of motion can, with the help of elementary calculus, determine the force that drives a planet on an elliptical orbit. If the same exercise is carried out on the epicyclic theory of copernicus, it becomes extremely complicated and by no means elegant. Nor does it lead to a simple driving force.

It has been the case so far in physics that simplicity and elegance appear to govern the basic laws of nature, although there is no clear understanding of why this should be so. While the laws themselves are simple and elegant, their consequences need not be. For example, a motion of three or more bodies in one another's gravitational attraction need not be simple or elegant as Laplace's Mecanique Celeste subsequently demonstrated. But the criteria of simplicity and elegance help when searching for the basic underlying principle. And this search was simplified for Newton by the hard work of Kepler spanning more than two decades.

But, why should gravity follow the inverse square law? This is where Newton stopped his quest – a quest that is still unended. Even today we do not know the answer in terms of anything 'more fundamental'. In this lecture I wish to highlight some major steps in this quest.

Mass and gravity

Let us examine the motion of a planet P under the force of gravity from the Sun. If $M_\odot$ is the Sun's mass that is exerting the force of gravity and $\bar{M}_P$ the mass of the planet that is responding to it then the force on the planet directed towards the Sun is

$$F_{PS} = G\frac{M_\odot \bar{M}_P}{r^2} \tag{1}$$

where r is the distance between the Sun and the planet and G the gravitational constant. Here we may call $M_\odot$ the *active* gravitational mass of the Sun and $\bar{M}_P$ the *passive* gravitational mass of the planet. If we inverted the roles of the Sun and the planet, the force on the Sun directed towards the planet will be

$$F_{SP} = G\frac{M_P \bar{M}_\odot}{r^2}. \tag{2}$$

Notice that the overhead bar has now gone to the Sun's gravitational mass which is passive while the planet's mass is the active one.

Newton's third law of motion acting across the distance r requires that $F_{PS} = F_{SP}$ and so

$$M_\odot \bar{M}_P = M_P \bar{M}_\odot,$$

$$\text{i.e.,}\qquad \frac{M_\odot}{\bar{M}_\odot} = \frac{M_P}{\bar{M}_P} = k \;\; \text{(constant).} \tag{3}$$

The result we have arrived at here tells us that the active and passive gravitational masses are proportional to each other and the constant of proportionality k is the same for all planets – in fact it has to be the same for all bodies. Therefore we may set it equal to unity by selecting suitable units for measuring these masses. So in terms of these units we have $M_P = \bar{M}_P$, $M_\odot = \bar{M}_\odot$ etc. and in essence there is really only one gravitational mass for each body.

Removing the overhead bar, we can now use Newton's second law of motion and write the acceleration of the planet towards the Sun as f_P where

$$m_P f_P = \frac{G M_P M_\odot}{r^2}. \tag{4}$$

Here m_P is the *inertial* mass of P.

A-priori it is not required that $m_P = M_P$. Indeed, if we consider the analogy of Coulomb's law governing the motion of an electron (e) attached by a proton (p) the corresponding equation is

$$m_e f_e = \frac{q_p q_e}{r^2}. \tag{5}$$

Here we do not have $m_e = q_e$, the electron charge. Likewise we do not have $m_p = q_p$, the proton charge. Indeed because the ratios q_e/m_e and q_p/m_p are so different that electrodynamics exhibits effects very different from gravity. In a given electric field a proton accelerates far less than an electron.

But in the case of gravity, as Galileo first showed, all bodies fall with equal rapidity in a given gravitational field. So, the ratio m_P/M_P

is the same for all bodies and again, with a suitable choice of units the inertial and gravitational masses can be made equal. It was this result that Einstein employed so cleverly to geometrize gravity.

For, it means that in the presence of gravity in any given region the world lines of all material particles will be the same. That is, the world lines are characterized by the 'geometry of spacetime', the geometry associated with the ambient gravity. So, the crucial result one needs is how these worldlines are determined. The Einstein equations give the prescription for determining the geometry and the rule is that in the absence of any other forces, the world lines are given by the geodesic principle

$$\delta \int ds = 0. \tag{6}$$

The operative phrase 'any other forces' sets gravity apart from the rest of physics; for the effect of gravity is already taken into consideration, by Einstein's equations. The equality of inertial and gravitational masses is the basis on which this facade rests.

Mach's principle

This intimate connection between inertia and gravity has led to a profound influence of theories of inertia on theories of gravity, general relativity included. The most important notion about inertia in this context is ascribed to Ernst Mach late in the nineteenth century.

Mach had been critical of Newton's laws of motion on the grounds that they are based on the physically unspecifiable concept of absolute space. The law of motion relating inertia m of a particle to its acceleration $\mathbf{f}$ under an impressed force $\mathbf{F}$

$$m\mathbf{f} = \mathbf{F} \tag{7}$$

is valid with respect to an abstract frame of reference which Newton designated as 'absolute space'. It is also valid in all other frames in uniform motion relative to it.

If, however, a reference frame is accelerated with respect to the absolute space, (7) is not valid. If the frame's acceleration is **a**, then to make (7) valid we have to make the following change :

$$\mathbf{F} \rightarrow \mathbf{F} - m\mathbf{a} \tag{8}$$

where the extra term now added depends on the inertial mass (m) of the particle and is termed 'inertial force'. The so-called centrifugal and Coriolis forces arising in the rotating frames of reference are examples of this fictitious force that needs to be incorporated if the second law of motion is to have the usual form in the accelerated frame.

If the give force **F** were uniform, we could find **a** such that the net force is zero in (8). This was the trick Einstein used to transform gravity away in his famous thought experiment of the freely falling lift. Again, we see that the experiment works because the Earth's gravitational force $\mathbf{F} = m\mathbf{g}$ also contains the same mass term. Thus $\mathbf{a} = \mathbf{g}$ is the solution valid for *all* material particles.

Returning to (8), and to Isaac Newton, the concept of absoluve space had given Newton considerable food for thought. That the inertial forces had tangible effects was demonstrated by him through his experiment of the rotating waterfilled bucket. Newton wrote :

> *'The effects which distinguish absolute from relative motion are, the forces of receding from the axis of circular motion. For there are no such forces in a circular motion purely relative, but in a true and absolute circular motion, they are*

greater or less, according to the quantity of the motion. If a vessel, hung by a long cord, is so often turned about that the cord is strongly twisted, then filled with water, and held at rest together with the water; thereupon, by the sudden action of another force, it is whirled about the contrary way, and while the cord is untwisting itself, the vessel continues for some time in this motion; the surface of the water will at first be plain, as before the vessel began to move; but after that, the vessel, by gradually communicating its motion to the water, will make it begin sensibly to revolve, and recede by little and little from the middle, and ascend to the sides of the vessel, forming itself into a concave figure (as I have experienced), and the swifter the motion becomes, the higher will the water rise, till at last, performing its revolutions in the same times with the vessel, it becomes relatively at rest in it. This ascent of the water shows its endeavour to recede from the axis of its motion; and the true and absolute circular motion of the water, which is here directly contrary to the relative, becomes known, and may be measured by this endeavour. At first, when the relative motion of the water in the vessel was greatest, it produced no endeavour to recede from the axis; the water showed no tendency to the circumference, not any ascent towards the sides of the vessel, but remained of a plain surface, and therefore its true circular motion had not yet begun. But afterwards, when the relative motion of the water had decreased, the ascent thereof towards the sides of the vessel proved its endeavour to recede from the axis; and this endeavour showed the real circular mo-

tion of the water continually increasing, till it had acquired its greatest quantity, when the water rested relatively in the vessel. And therefore this endeavour does not depend upon any translation of the water in respect of the ambient bodies, nor can true circular motion be defined by such translation. There is only one real circular motion of any one revolving body, corresponding to only one power of endeavouring to recede from its axis of motion, as its proper and adequate effect; but relative motions, in one and the same body, are innumerable, according to the various relations it bears to external bodies, and, like other relations, are altogether destitute of any real effect, any otherwise than they may perhaps partake of that one only true motion.'

We can sum up Newton's interpretation of his experiment by saying that absolute rotation has nothing to do with the relative rotations which are directly observed, and that, nevertheless, we *can* determine experimentally the amount of absolute rotation possessed by a body. All we have to do is to measure the curvature of a water surface rotating with the body.

So the absolute space appears to have real existence; but there does not seem any independent way of identifying it. Earnst Mach provided that missing link, which I will describe next.

Let us consider an investigation that can be carried out in two ways to answer the following question : 'What is the spin of the earth around its polar axis?'

The astronomer would answer it by noticing that the stars rise and set and rise again – a phenomenon that repeats itself every 24 hours (or nearly so!). Thus he can identify that period with the earth's spin period. This observation can be improved further by allowing for a) the annual rotation of the earth round the Sun and b) the rotation of the Sun round the Galactic Centre.

The laboratory physicist can answer the question by setting up a Foucault pendulum. The fact that the pendulum's oscillations are being measured in a reference frame fixed on the spinning earth gives rise to inertial forces of which the most effective is the Coriolis force. Because of this force the plane of oscillation of the pendulum turns round the local vertical with a period.

$$T = T_0 \operatorname{cosec} l, \tag{9}$$

where l is the local latitude and T_0 the spin period.

Both the laboratory and the astronomical methods give the same value for the answer. This fact is often dismissed by the comment 'so what?'. The comment is not justified when you remember that both the astronomer and the laboratory physicist are measuring different quantities. The former is measuring the spin period relative to the distant cosmic background while the latter is measuring it relative to Newton's absolute space.

Ernst Mach highlighted this issue and stressed that the fact that two methods agree gives us an important information : namely, that the distant cosmic background *coincides* with Newton's absolute space.

Here, argued Mach, is the missing link that allows us to concretise the abstract notion of absolute space. It tells us that the absolute space (more correctly the *local* inertial frame) is *determined* by the cosmic background. Mach went even further and suggested that because the very notion of inertia and its quantitative measure, the mass, depend on this special reference frame, there is a link between the very large scale structure of the universe and the property of inertia. He wrote in 1872 :

> *'For me only relative motions exist. ... When a body rotates relatively to the fixed stars, centrifugal forces are produced; when it rotates relatively to some different body and not relative to the fixed stars, no centrifugal forces are produced. I have no objection to just calling the first rotation so long as it be remembered that nothing is meant except relative rotation with respect to the fixed stars.'*

He further said :

> *'Obviously it does not matter if we think of the earth as turning round on its axis, or at rest while the fixed stars revolve round it. Geometrically these are exactly the same case of a relative rotation of the earth and the fixed stars with respect to one another. But if we think of the earth at rest and the fixed stars revolving round it, there is no flattening of the earth, no Foucault's experiment, and so on–at least according to our usual conception of the law of inertia. Now one can solve the difficulty in two ways. Either all motion is absolute, or our law of inertia is wrongly expressed. I prefer the second way. The law of inertia must be so conceived that exactly the same thing results from the second supposition as from the first. By this it will be evident that in its expression, regard must be paid to the masses of the universe.'*

Mach's principle quantified

Einstein, a onetime pupil of Mach, was impressed by this chain of reasoning and hoped that his theory of gravity would turn out to incorporate Mach's principle. This hope was not realized in the end. There are several anti–Machian solutions in general relativity.

For example, there are empty space solutions that are nontrivially different from the flat spacetime of special relativity. In these solutions $R_{ik} = 0$ but $R_{iklm} \neq 0$. What do the timelike geodesics in such spacetime mean? With no 'background' of matter why are these trajectories of 'particles under no force' singled out?

On a second count there are cosmological solutions of Einstein's equations wherein the distant background *rotates* with respect to the local inertial frame. Ironically, the classic paper of Kurt Gödel[1] which produced one such model appeared in the 70th birthday festschrift for Einstein. By then, however, Einstein himself had lost his enthusiasm for Mach's principle. In his autobiographical notes he writes[2] :

> *'Mach conjectures that in a truly rational theory inertia would have to depend upon the interaction of the masses, precisely as was true for Newton's other forces, a conception which for a long time I considered as in principle the correct one. It presupposes implicitly, however, that the basic theory should be of the general type of Newton's mechanics : masses and their interaction as the original concepts. The attempt at such a solution does not fit into a consistent field theory, as will be immediately recognized.'*

There were others, however, who felt that Mach's principle needed to be incorporated in a theory of gravity. For example, Dennis Sciama[3] in the mid-fifties suggested that the observed 'coincidence'

$$\rho G \tau^2 \sim 1 \qquad (10)$$

where $\rho =$ mean density of matter in the universe and $\tau \sim$ characteristic time scale of expansion of the universe as measured by Hubble's constant, is a reflection of Mach's principle. For instance Sciama wrote :

> *'Our ability to calculate this average density is not surprising, once we understand the significance of the relation between the gravitational constant G and the amount of matter in the stars. This significance can be expressed as follows. We saw when we defined the gravitational constant G that it is a measure of the gravitational force produced by a body of given inertial mass. We can reverse this, and say that G* is a measure of the inertial mass of a body which produces a given gravitational force. Now in the present theory, a body produces the same gravitational force whatever other bodies there are in the universe. On the other hand, its inertial mass is induced into it by all these other bodies. Hence G, which measures the ratio of inertial mass to gravitational mass, is determined by these bodies. Our formula for G, the density of matter, and Hubble's constant, is just the mathematical expression of this physical relationship.'*

In 1961, Carl Brans and Robert Dicke[4] proposed another version of Mach's principle. If we write M as the mass of the observable universe and R its range then (10) is equivalent to

$$\frac{M}{Rc^2} \sim G^{-1} \tag{11}$$

Brans and Dicke interpreted (11) as a crude approximation of

$$G^{-1} \sim \sum_i \frac{m_i}{R_i c^2} \tag{12}$$

where m_i is a mass at distance R_i from the observer, there being $i = 1, 2, \ldots$ masses in the universe. Written in this form G^{-1} appears as a scalar field ϕ whose sources are in masses. In other words, the very strength of gravity is related to inertia through ϕ.

Known as the scalar–tensor theory (because of the scalar ϕ and the usual second rank tensor R_{ik} of relativity both being incorporated in it) the Brans–Dicke theory played an important role as an alternative to relativity, thus prompting dramatic improvements in the observational tests of the two theories within the solar system. Though largely discredited now by these very tests, the theory has to be accorded its due credit for its contributions to our understanding of gravity.

In 1964, Fred Hoyle and I proposed an action-at-a-distance theory of inertia which directly incorporated Mach's principle[5]. In this theory the inertial mass of a^{th} particle ($a = 1, 2, \ldots$) at world point X was given by

$$m_a(X) = \lambda_a \sum_{b \neq a} \lambda_b \int G(X, B) ds_b \tag{13}$$

where ds_b is the element of proper time on the worldline of particle b and λ_b a coupling constant. The action at a distance is through the two-point scalar propagator G satisfying the relations.

$$\Box G(X, B) + \frac{1}{6} R G(X, B) = \delta_4(X, B)/\sqrt{-g}(X), \tag{14}$$

and

$$G(X,B) = G(B,X), \quad (15)$$

where $\square$ and R are evaluated at X.

I will not go into details of this formulation except to highlight some special features.

(i) The dynamics of the theory are derived from an action principle

$$\delta \sum_a \sum_{\neq b} \int \int \lambda_a \lambda_b G(A,B) ds_A ds_b = 0. \quad (16)$$

(ii) The theory is conformally invariant with the proviso that there exist certain hypersurfaces of zero mass. If one insisted on transforming to a conformal frame in which all masses are constant and non zero, one would arrive at Einstein's equations with $G > 0$ and with spacetime singularities on the zero mass surfaces.

(iii) If the zero mass surface has kinks then it may be possible to interpret them as describing newly created matter and to use this idea to explain the anomalous redshifts of quasars and galaxies.

Some versions of the theory also predicted a slow variation of G with time. Recent radar and laser ranging experiments on the planets and the Moon, however, appear to rule out $|\dot{G}/G| \geq 10^{-11}$ yr^{-1}.

I mention these examples to illustrate how a philosophical idea that originated with Mach has led to tangible observational developments in the field of gravitation and cosmology.

Gravity as a part of the unification programme

I turn now to the more fundamental side of gravity and to the deeper question of how it can be unified with the rest of physics. Although Einstein himself unsuccessfully attempted unification, paradoxically his approach to gravity has been detrimental to the unification programme. Let me clarify this statement.

By geometrizing gravity Einstein effectively removed it from the scene as a force. The existence of a force in general relativity is detected through nongeodetic motion. For example an electron in the neigbhourhood of the Sun would move along a timelike geodesic if there were no magnetic field in the Sun. Because the Sun has a magnetic field the electron would not move along the geodesic. Thus geometrisation of these trajectories is not naturally achievable.

If one were to unify gravity with electromagnetism, along Einstein's line then it is necessary to transform the former also into a geometric entity. This, however, is not so easy even by enlarging the spacetime to higher dimensions. For, unlike Galileo's heavy and light bodies, the electron and the positron do not have the same trajectories in a given electro magnetic field despite having the same mass. In general the ratio e/m (charge/mass) enters explicitly into the equations of motion of an electric charge.

If, however, we retain the electromagnetic interaction as a separate force we cannot naturally unify it with gravity which is no longer a force. This has been one reason why the unification programme for basic physical interactions has proceeded in a manner complementary to that attempted by Einstein–by uniting the other basic interactions and keeping gravity out.

In the so called 'grand unified theories' (GUTs) the strong interaction is sought to be united with the weak and the electromagnetic interactions. The latter two can be unified as the 'electroweak theory' by the Weinberg–Salam approach of gauge theories. The GUTs therefore seek the unification that has a larger gauge group of symmetry that includes the $SU(3)$ group of strong interactions as well as the $SU(2)_L \times U(1)$ group of the electroweak theory.

It is still too early to predict whether this approach will succeed. The encouraging signs are that gauge theories are renormalizable and so one may hope for a unified theory that allows perturbation expansions to be carried out. However, so far as gravity is concerned this creates an additional barrier between the GUTs and gravity : for gravity as developed by Einstein is not a gauge theory in the usual sense of the word. Indeed, as shown by Richard Feynman in the early sixties, the first order tensor field theory of spin 2 that one obtains by linearizing general relativity is not renormalizable.

Quantum gravity

Since unification is ultimately to be achieved at the quantum level the theoretician is inevitably driven to the job of quantizing gravity. Note that unlike other interactions where laboratory or cosmic ray observations forced one to consider quantum theories of those interactions, here we have *no* experiment that makes it necessary to think of quantum gravity.

The transition of rules governing a system from classical to quantum physics is determined by the ratio $J/\hbar$ – the ratio of the action of the system to Planck's constant (divided by 2π). Let me illustrate with the help of electrodynamics.

An electron of mass m, charge $(-e)$ moving in the field of a proton of mass $M \gg m$ and charge $(+e)$ has the Lagrangian

$$L = \frac{1}{2} m \dot{\mathbf{r}}^2 + \frac{e^2}{r}. \tag{17}$$

Suppose the electron moves in a circular orbit of radius R_0. Then we have for this orbit

$$\frac{m|\dot{\mathbf{r}}^2|}{r} = \frac{e^2}{r^2},$$

$$\text{i.e.,} \quad L = \frac{3e^2}{2R_0}. \tag{18}$$

The characteristic time of the orbit is

$$T = \frac{2\pi R_0}{|\dot{\mathbf{r}}|} = \frac{2\pi R^{3/2} m^{1/2}}{e}. \tag{19}$$

Multiplying (18) with (19) we get the classical action for this motion as

$$J \sim 3\pi e R^{1/2} m^{1/2} \tag{20}$$

Now, our criterion says that the above calculation based on classical mechanics is valid provided $J/\hbar \gg 1$, i.e., if

$$R_0 \gg \frac{\hbar^2}{me^2} \sim 10^{-10} m. \tag{21}$$

The right hand side is of the order of the order of atomic dimensions, thus telling us that our classical mechanics breaks down when discussing the electrodynamics of the atomic systems. Whenever $J/\hbar \sim 1$, the rules change to those of quantum mechanics.

Let us now examine the corresponding situation for gravity. If we consider Newtonian gravity, then the result (21) is altered to

$$R_0 \gg \frac{\hbar^2}{Gm^2 M} \sim 10^{-50} m. \tag{22}$$

where we have replaced e^2 by GmM.

This limit illustrates how ridiculously small in size the system has to be in order to show quantum effects of Newtonian gravity.

Nevertheless we may improve the situation somewhat by going to relativity. For example, the orbital velocity at $10^{-50}m$ far exceeds the speed of light! So to bring in general relativity we consider the Hilbert action

$$J = \frac{c^3}{16\pi G} \int R\sqrt{-g}d^4x. \tag{23}$$

Over a characteristic length scale set by

$$L \sim R^{-1/2} \tag{24}$$

the above action becomes

$$J \sim \frac{c^3}{G} L^2 \tag{25}$$

so that the classical validity extends to

$$L \gg \sqrt{\frac{G\hbar}{c^3}} \equiv L_P \sim 10^{-35}m. \tag{26}$$

where L_P is the 'Planck length.'

Note that the classical to quantum criterion for electrodyunamics contained the electron mass explicitly in the length scale. The Planck length *does not* depend on any such particle mass. This length is characteristic of spacetime geometry and as such applies to *any* particle moving in it. The limit in (26) is considerably higher than the Newtonian limit (22) as was to be expected in view of the light speed limit operating in relativity.

But what should one make of this result? To the extent that it tells us not to trust the results of classical general relativity down to this low limit of L_P it serves a useful purpose. But what laws of quantum gravitomechanics should one use instead? This question brings us in head-on collision with the wall of quantum gravity.

For, there are numerous conceptual and operational difficulties inherent in this wall. I enumerate them briefly. The operational difficulties may be somehow circumvented but the conceptual ones are not to easy.

The first conceptual difficulty is in the basic question : "What are we quantizing?" Recall that in ordinary situations involving other interactions, like quantum electrodynamics, the motions of particles and the electromagnetic interactions themselves are quantized. While we could still quantize particle trajectories in a given curved spacetime, how do we quantize the gravitational interaction? For there is no longer a 'force of gravity' to quantize : it has been replaced by the non-Euclidean spacetime geometry.

The second conceptual problem comes when we try to quantize geometry. In normal quantization of any field the background spacetime is given : in fact it is Minkowskian in the field theories discussed by particle physicists. Here, however, we are out to quantize the spacetime itself.

Amongst the operational problems may be mentioned the nonlinearity and nonrenormalizability of general relativity. It may be possible to circumvent the latter by using a nonperturbative field theory approach. The nonlinearity may be handled by using some new variables. The approach currently pioneered by Abhay Ashtekar[6] seems a promising step in that direction.

Assuming that the holy grail of quantum gravity is eventually attained, the question still remains : "What practical dividends will it pay?" Except for the state of the univese very close to big bang there is no phenomenon known today that forces us to think of modifications at the level of L_P. I therefore end by discussing this particular era in the history of the big bang universe.

Quantum cosmology

The big bang cosmological model as given by the classical general relativity has the spacetime described by the line element

$$ds^2 = c^2dt^2 - S^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right\} \qquad (27)$$

where $S(t)$ is the expansion factor. The epoch $t = 0$ is the so called 'big bang' epoch. k is the curvature parameter with values $0, 1$ or -1.

This classical model suffers from three defects peculiar to non-Euclidean geometry : the spacetime singularity at $t = 0$, the particle horizon problem and the flatness problem. Let me briefly discuss them.

The spacetime singularity arises from the vanishing of $S(t)$ at $t = 0$. At that epoch the geometrical parameters either vanish or blow up, thus leading to a breakdown of the principle of equivalence. No remedy for this defect is available within the classical relativity unless one is willing to give up some 'reasonable energy conditions'.

The function $S(t)$ behaves like $t^{1/2}$ for small t and this introduces particle horizons of size $\sim 2ct$. For small t the horizons are so small that the univese is split into a large number of small causally disconnected regions, each region being internally casually connected. So the question arises : why does the universe appear so homogeneous on a large scale as evidenced by the smoothness of the microwave background?

The flatness problem relates to the parameter k. For $k = +1$ the universe is closed and it collapses eventually to a zero value of $S(t)$. For $k = -1$ the universe is open and it expands out to infinity. Now the time scale for this to happen is not given by the theory but depends on the initial conditions. If one looks at the very early epochs, the characteristic time scales are very small, ranging from the Planck time scale $L_P/c \sim 10^{-43}s$ to the time scale $\sim 10^{-36}s$ characteristic of grand unification of theories. If the initial conditions of the universe were determined during this era, it becomes a mystery as to how it is still expanding after a time scale of $\sim 3 \times 10^{17}s$: why has it not collapsed or blown apart to infinity? This is possible only if the universe were very finely tuned to values of density and curvature that correspond to the 'flat' spatial model $k = 0$. Why this early preference for a flat or nearly flat model?

To answer these questions we might resort to quantum gravity as one of the ways. In fact we might *demand* the resolution of these problems as the prerequisite of a successful theory of quantum gravity. So far, however, none of the formal approaches to quantum gravity has advanced to a state where it can throw light on these problems.

I will describe a pragmatic and half baked approach that has the merit of tackling these problems *within* its somewhat restricted framework. This is the approach of conformal quantisation. To introduce it I will first discuss conformal transoformations.

We may consider two manifolds $\bar{\mathcal{M}}$ and $\mathcal{M}$ with metrics related by

$$\bar{g}_{ik} = \Omega^2 g_{ik} \tag{28}$$

where Ω is an arbitrary function of spacetime coordinates x^i, $i =$

0, 1, 2, 3. We say that $\bar{\mathcal{M}}$ is a conformal transform of $\mathcal{M}$. This transformation preserves the global light cone structure and hence all causal relationships in spacetime. In any quantum gravity transition from $\mathcal{M}$ to $\bar{\mathcal{M}}$ these relationships are preserved.

We may state this more strongly : the most general transformation of geometry that preserves the global causality has to be a conformal transformation.

Do nonconformal transformations have a *locus-standi* at all in quantum gravity? If they do, then they will play havoc with physical causality. The spacetime points causally connected in $\mathcal{M}$ may no longer be so in $\bar{\mathcal{M}}$ and vice versa. How to handle such scenario is the problem all formal approaches to quantum gravity have to face. We will skirt round it and stick to conformal transformations only – more in the spirit of looking for the lost needle only in lighted corners.

The simplicity of this restriction is that the problem of quantum gravity becomes explicitly solved. Consider the problem as follows.

Foliate the spacetime $\mathcal{M}$ with spacelike hypersurfaces $\sum$, given by x° = constant. On each $\sum$ we have the 3-geometry given by $G^{(3)}$, say. The classical geometrodynamics of Einstein's relativity tells us how to compute $G_2^{(3)}$ on Σ_2 given $G_1^{(3)}$ on an earlier Σ_1. As discussed by Isenberg and Wheeler[7], the conformal part of $G_1^{(3)}$ and the extrinsic curvature of Σ_1 are needed to specify the initial value problem.

In quantum cosmology we formulate a different problem. We attempt to construct a propagator that determines the probability amplitude that given $G_1^{(3)}$ on Σ_1 we will find $G_2^{(3)}$ on Σ_2. This amplitude is given by the Feynman path integral

$$K(\Sigma_2, G_2^{(3)}; \Sigma_1, G_1^{(3)}) = \int \exp\,(iJ/\hbar)\mathcal{D}\Gamma \tag{29}$$

where J is the classical action for gravity and Γ a typical path in the space of 3-geometries that begins at $G_1^{(3)}$ on Σ_1 and ends at $G_2^{(3)}$ on Σ_2.

So far we have been quite general. Had it been possible to evaluate the above path integral or even to manipulate with it in simple cases, the problem of quantizing gravity would have been solved. Unfortunately reality is otherwise. One needs to simplify the matter further by restricting to fewer degrees of freedom than the ∞^6 contained in the space of all 3-geometries.

A drastic simplification is to confine attention to only the conformal degrees of freedom, i.e., to metrics of the form $\Omega^2 g_{ik}$ where g_{ik} is the metric of a classical solution of Einstein's equations. So we talk of a propagator K which takes a geometry with conformal function Ω_1 on Σ_1 to another with Ω_2 on Σ_2.

This problem is therefore one of quantum conformal fluctuations. It is solvable exactly because the path integral is one of quadratic exponential in Ω. I will not go into details of the computations except to say that $K(\Sigma_2, \Omega; \Sigma_1, \Omega_1)$ can be explicitly evaluated.[8]

Returning to the classical Friedman solution, we consider any conformal transform of it by using an Ω that is an arbitrary function of t only. This preserves the cosmological principle although the new spacetime is no longer a solution of Einstein's equations. Each $\Omega \neq$ constant therefore defines a non-classical path Γ in the space of geometries, with Γ_c being the classical path $\Omega =$ constant. We may define the final state of the universe at $t = t_2$ by $\Omega = \Omega_2$ and assume that for $t \gg t_2$ the universe is almost classical, i.e., $J \gg \hbar$. We do not know what state the universe was in at $t = t_1 \ll t_2$ when $J \leq \hbar$. So we write the evolution of the universe between t_1 and t_2 by the quantum relation

$$\psi_1(t_1, \Omega_1) = \int K(t_2, \Omega_2; t_1, \Omega_1)\psi_2(t_2, \Omega_2)d\Omega_2 \tag{30}$$

where ψ_1, ψ_2 are the wave functions of the universe at t_1 and t_2. The function ψ_2 has a compact support around a constant value $\Omega_2 = \Omega_0$, say. For example, it may be a wavepacket centred on Ω_0 with a small dispersion σ_2.

The interesting result that emerges is that although ψ_1 is also centred around $\Omega_1 = \Omega_0$, its dispersion σ_1 is much larger. In fact, $\sigma_1 \to \infty$ as $t_1 \to 0$. In other words, the quantum uncertainty about the state of the universe in the past grows indefinitely as the past epoch approaches the classical singular epoch. Moreover, whether the universe was singular or not at $t = 0$ can be answered in terms of quantum probabilities : it turns out that the set of singular solutions has the probability measure zero.

This work can be generalized to the conformal function Ω depending on space as well as time and the classical solution being *any* singular solution of Einstein's equations. The result of singular non-classical solutions having a set of measure zero in probability continues to hold. With the singularity (almost) gone the particle horizon is also unlikely to survive and thus both the singularity and horizon problems are cured.[9]

So far as the flatness problem is concerned the result is stated thus. If we assume that the universe was initially in the empty Minkowski form then it is unstable to conformal fluctuations. We can determine the probability of its transition to any conformally flat form : however, the relative probability is overwhelmingly large for Ω a function of time only. This is none other than the case of a

Robertson–Walker spacetime with $k = 0$. Thus it is very likely that the universe made a transition to the $k = 0$ model provided it was initially empty with Minkowski spacetime.[10]

The interesting result to explore is whether as the 'next best' alternative to the smooth $k = 0$ R–W model we have small space dependent perturbations on Ω and if so whether these perturbations act as seeds for galaxies.

Concluding Remarks

This approach illustrates that much can be achieved in quantum cosmology if one approaches it from the hiuristic rather than the formal end. We may recall that quantum theory itself grew in the hiuristic fashion and even today it has several epistemological issues to clear up. If one had insisted on first creating a formally satisfactory framework of quantum theory before trusting its results for specific problems, we would not even have solved today the problem of the hydrogen atom!

To end this talk I return to Newton's "hypotheses non fingo". I believe it is still the right attitude to take towards gravity which continues to be the most mysterious of all physical interactions. While developing formal structures has its own attractions, the proof of the pudding lies in the eating. Today we are concentrating more on making formal recepies for quantum gravity–recipes that look attractive to read in a cook book but which no cook can ever translate into a tangible pudding. Is it not high time to take a pan and begin cooking, howsoever primitive the recipe may look?

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