Low energy theorems for electromagnetic and gravitational radiation

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The trinity

He (PCV) urged the use of both parts of the brain - left and right; left for analytical and mathematical, and right side for physical significance.— ICGC 1987.

Background...

- C.F. Cho and NDH(Ann.Phy. 96, p406,1976) were the first to give the correct classical derivation of the Gravitational Spin Precession in this system.
- The first GR derivation was given by Börner, J. Ehlers and E. Rudolph(Astron. Astrophys 44 p.417(1975).
- The expression that is widely quoted in literature now came out of the calculation of the gravitational interaction between two Dirac fermions by Barker and O'Connell(ApJ 199, L25, 1975).
- V. Radhakrishnan and NDH(Astrophysical Letters Vol 16, p.135, 1976) were the first to give a detailed method to observe this effect based on pulse profiles.
- Their analysis predicted in detail the effects of spin precession in pulsars both on pulse profile as well as the polarisation sweep.

New controversy

- Ehlers, Rosenblum, Goldberg and Havas in 1976 (Ap.J.Letts 208(1976) L77) criticised the Einstein derivation on a number of counts.
- Einstein had been rather clumsy in his derivations.
- For all those reasons, Ehlers et al advocated a critical re-look at this very important issue.
- Among the items they recommended for a careful treatment was the issue of the so-called logarthmic deviation of light-cones even in the so called radiation zone(more on this later)
- In 1978 Rosenblum claimed that one needs 3rd order post-newtonian approximation, and further claimed the correct answer to be 2.5 times the Einstein result.

Our approach I

- I first describe work done with V. Soni (*Feynman graph derivation of the Einstein quadrupole formula*, J.Phys. A, Math and Gen.,15(1982) 473-492) that was based on a Feynman graph approach in spin-2(rather, helicity-2) theories of gravitation.
- Normally one would not resort to quantum methods to resolve admittedly classical issues, though there is nothing wrong in principle in doing so.
- One might even argue that philosophically that is the correct way as the world is quantum mechanical!
- In the present case, the main motivation came from the fact that even careful classical calculations had come under a shadow.
- Some obvious advantages of the Feynman graph approaches are that particle equations of motion are automatically taken into account through conservation laws at the vertices.

Feynman graph..

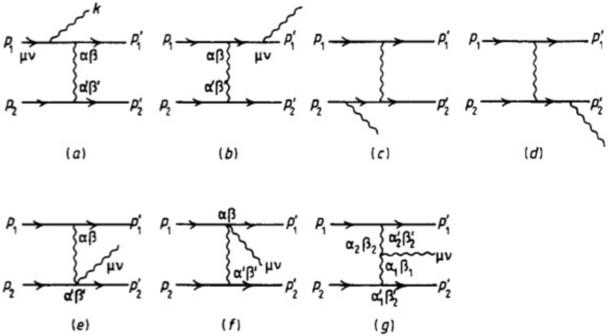
- General coordinate invariance translates to the better understood and computationally easier gauge invariance.
- Many valid objections can also be raised at this point.
- The most serious could be that no fully satisfactory quantum gravity theory has yet been found(this remains true even 40 years after my work!).
- My response to that is that we are ultimately interested in the classical limit and this is captured by the tree graphs for which there are consistent and satisfactory treatments.
- In fact, regularised perturbative quantum gravity is fully under control.
- In the limit one is interested in, renormalisation problems do not arise.
- No need to struggle with subtleties and nuances of the pseudo tensor

Feynman...

- Another serious issue is the presence of the potentially large parameter $Gm_1 m_2$.
- As it turns out, this is tied up with the other serious lacuna of massless field theories i.e the lack of a strict S-matrix.
- In plainer terms this has to do with the fact that even at very large distances, plane waves are not a solution.
- This is quite familiar and well understood, for example, in the solutions of the Schrödinger equation for Coulomb potentials.
- Coulomb distorted wave functions
- It turns out that the precise analog of this in GR is the logarithmic distortion of distant light cones.

Gravitational Scattering Case

- The diagrams contributing to gravitational radiation during gravitational scattering are shown below.
- It is to be noted that all these are of order κ^3.
- The criticism of Einstein's original derivation becomes self-evident here.



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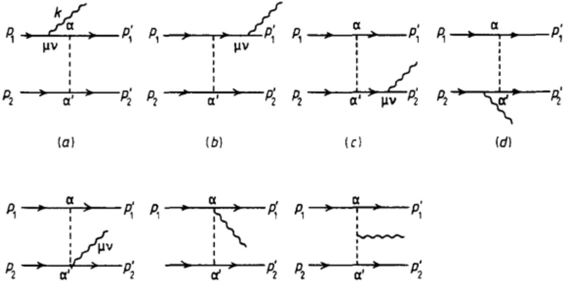

Non-relativistic reductions

- We leave out the details and present results for the amplitudes in the non-relativistic limit:
- The result for gravitational scattering case is:

$$M_{ij}^{g} = 4\,i\,\kappa^{3}\,\frac{m_1\,m_2}{q^2}\{\frac{M}{8\omega}(p^i p^j - p'^i p'^j) + \mu\,M\,\frac{q^i q^j}{q^2}\}$$

- The result for electromagnetic scattering case is:

$$M_{ij}^{em} = -8\,i\,\kappa\,\frac{m_1 + m_2}{q^2}\,e_1\,e_2\{\frac{M}{8\omega}(p^i p^j - p'^i p'^j) + \mu\,M\,\frac{q^i q^j}{q^2}\}$$

- It is to be noted that these are simply proportional to each other!
- This hints to a universal form for gravitational radiation.

Coulomb distorted Born approximation

- The final result when long range modifications to plane waves is taken into account: The two particle state becomes

$$\chi(\vec{r}_1, \vec{r}_2, t) \;=\; e^{i(E_1+E_2)t}\, e^{i\vec{P}\cdot\vec{R}-\frac{i}{2}\,\vec{p}\cdot\vec{r}}\, F(i\nu, \frac{ipr}{2} - \frac{i}{2}\vec{p}\cdot\vec{r})$$

$$M_{ij} = \iint d\mathbf{q}\, d\mathbf{q}'\, \psi^*_-(-i\nu', \mathbf{p}', \mathbf{q}')\, \{\ldots\}\psi_+(i\nu, \mathbf{p}, \mathbf{q})$$

- Here $\nu = \frac{Gm_1 m_2 \mu}{p}$ for the gravitational case, and

$$\psi(i\nu, \mathbf{p}, \mathbf{q}) = \frac{8\pi\nu\, p}{(p^2 - q^2)^{1+i\nu}}\, \frac{1}{|\mathbf{p} - \mathbf{q}|^{\mathbf{2-2i\nu}}}$$

Logarthmic distortion of light cones

- The graviton wave function in the radiation zone approaches the form:

$$h_{\mu\nu} \rightarrow \epsilon_{\mu\nu}\, e^{i\omega\, t - i\vec{k}\cdot\vec{r} + 2iGM\omega \ln r}$$

- Clearly the light cones are logarithmically distorted w.r.t flat light cones!
- This was one of the issues that Ehlers et al had emphasised.
- Suggests the new time coordinate

$$t^* = t + \frac{2GM}{c^3} \ln r$$

- This is the same as what Anderson had used to analyse this issue!
- Most remarkably, the long range modifications to the graviton wavefunction do not affect the matrix elements.

One Graviton Transition Operators

- The final results can be cast in the form

$$M_{ij} \rightarrow \frac{-4m_1 m_2}{\omega} \{\frac{\kappa}{2}\dddot{D}_{ij}\epsilon^{ij}\}_{fi}$$

- This allows one to identify

$$\{\frac{\kappa}{2}\dddot{D}_{ij}\,\epsilon^{ij}\}$$

  as the one graviton transition operator

- For the electromagnetic radiation this, in the leading order, turns out to be

$$M_{ij} = -\frac{4m_1 m_2}{\omega}\{\vec{\epsilon}\cdot\ddot{\mathbf{d}}\}_{fi}$$

- On comparing the Larmor power formula there, one immediately arrives at the Einstein quadrupole formula!!

Approach II: Low Energy Theorem Approach

- *Low energy theorems for quadrupole radiation in electromagnetism and gravitation*, N.D. Hari Dass,NBI-HE-81-45 (1980) (Unpublished).
- Low energy theorems are powerful techniques pioneered by particle physicists when they were desperately groping for a theory of strong interactions.
- Pioneered by Nambu, Schwinger, Weinberg, Mandelstam, Francis Low and others, these theorems allowed one to understand many aspects of hadrons even in the absence of a fundamental or even effective theory for them.
- The earliest example was Low's theorem for Compton Scattering which showed that at very low frequencies the scattering was entirely determined by the total electric charge, and nothing else(in the sense that structural details were irrelevant).

Low Energy Theorems...

- Next such, due to Low again, showed that power radiated through electromagnetic radiation was entirely governed by the time rate of change(actually second derivative) of the electric dipole moment and nothing else.
- In fact, in my application of these techniques to gravitational radiation, I closely followed Low.
- As a spin-off, I found that Low's treatment gives more even for electromagnetic radiation!(Though Low also had claimed this he had omitted some crucial terms needed for this).
- The next correction is also universal, given by third time derivative of quadrupole moment.
- A vast majority of processes involving mesons and photons could be described in terms of a few parameters long before QCD came on the scene!

Electromagnetic and Gravitational LET's: Weinberg

- Weinberg in the 1960's pioneered the study of gravitational LET's. A remarkable result proved by him was:for arbitrary processes

$$A_1 + A_2 + A_3 + \ldots \to B_1 + B_2 + B_3 + \ldots$$

gauge invariance of amplitude to emit a soft photon requires

$$\sum q_i = \sum q_f$$

However, the analogous gravitational LET yields

$$\sum \kappa_i P_i^\mu = \sum \kappa_j P'^\mu_j$$

Conservation of energy-momentum

$$\sum P_i^\mu = \sum P'^\mu_j$$

immediately implies

$$\kappa_i = \kappa$$

This is the Principle of Equivalence!

- Then came the Hulse-Taylor pulsar in late 1974. The test particle assumptions broke down.
- Barker and O'Connell had proposed a formula for spin-precession based on a calculation of the gravitational interaction between two Dirac particles.
- With C.F. Cho I calculated this purely classically using Schwinger Source Theory. The answer agreed with Barker-O'Connell result!
- I again smelled Low Energy Theorems!

Gravitational LET's..

- We (C.F. Cho and NDH, Phys.Rev. D14, p.2511, 1976) proved a LET for the stress tensor of the form:

$$\langle \mathbf{p}'|T^{\mu\nu}|\mathbf{p}\rangle = \kappa\{(\mathbf{p}+\mathbf{p}')^{\mu}(\mathbf{p}+\mathbf{p}')^{\nu} + q^{\mu}\Sigma_{\mu\nu}p^{\nu} + \mu \leftrightharpoons \nu\}$$

- This is a completion of Weinberg's results to include spin(this had also been derived independently by Deser and Boulware shortly before).
- Because of the inherent ambiguities of the stress-tensor, this is all that one can achieve.
- This single LET correctly accounted for all the classical predictions of GR (excluding perihelion advance and gravitational radiation)!
- It also correctly reproduces many one graviton exchange quantum processes.
- In electromagnetism the corresponding LET is:

$$\langle \mathbf{p}'|J^{\mu}|\mathbf{p}\rangle = 2ep^{\mu} + O(q) + \ldots$$

How to derive LET's for radiation?

- One starts with T-matrices for vertices and for scattering.
- T-matrix becomes the S-matrix when all external legs go on shell
- S-matrices are observable
- When the emitted radiation is very soft, except for the leg connecting the scattering T-matrix to the vertex T-matrix, all others are on-shell.
- The exceptional leg is only mildly off-shell because of the softness of the radiation. Hence the T-matrices can be Taylor expanded.
- The sum of all diagrams where radiation is from the external legs is not gauge invariant ingeneral.
- One adds the internal bremsstrahlung diagram to restore gauge invariance.
- The sum of all terms now give the LET amplitudes.

LET vertices

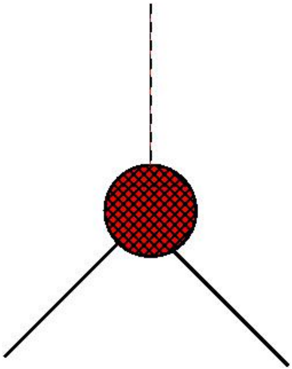

Figure: The vertex T-matrix

LET scattering T-matrix

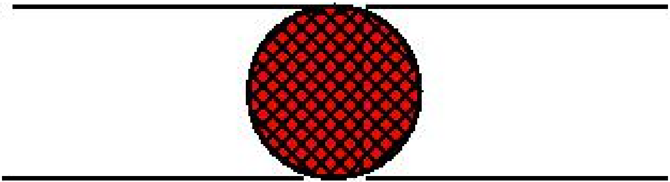

Figure: Scattering T-matrix

$$T(p_1^2, p_2^2, {p'_1}^2, {p'_2}^2, \nu, \Delta)$$

$$\nu = p_1 \cdot p_2 + p'_1 \cdot p'_2 \qquad \Delta = \mu\{ \frac{(p'_1 - p_1)^2}{m_1} + \frac{(p'_2 - p_2)^2}{m_2} \}$$

$$q = p'_1 - p_1 = p_2 - p'_2$$

LET for radiation

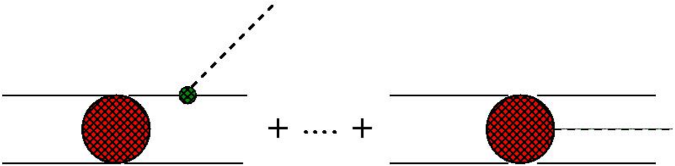

Figure: Radiation LET's

Some details..

- Consider the first of the diagrams. Its contribution is

$$M_\mu = \frac{2e_1\, {p'_1}_\mu}{2\, p'_1 \cdot k}\, T(m_1^2, m_1^2 + 2\, p'_1 \cdot k, m_2^2, m_2^2, \nu + p'_2 \cdot k, \Delta + \frac{2\mu}{m_1}\, k \cdot q)$$

$$M_\mu = \frac{e_1\, {p'_1}_\mu}{p'_1 \cdot k}\, \{T_0 + 2\, p'_1 \cdot k\, T_2 + 2p'_2 \cdot k\, T_5 + \frac{2\mu}{m_1}\, k \cdot q\, T_6 + \ldots\}$$

$$k \cdot M = e_1 \{T_0 \ldots\}$$

- Adding all such contributions one finds T_0 terms cancel completely(Weinberg's electro LET).
- For the first diagram find $\tilde{M}_\mu$ such that

$$k \cdot M + \tilde{M} = 0$$

$$M_\mu = e_1 \{\frac{{p'_1}_\mu}{p'_1 \cdot k}\, T_0 + {p'_1}_\mu\, \frac{p'_2 \cdot k}{p^{\cdot}_1 k}\, T_5 + \frac{2\mu}{m_1}\, k \cdot q\, \frac{{p'_1}_\mu}{p'_1 \cdot k}\}$$

Final amplitude: electromagnetic case

$$
\begin{aligned}
M^{\mu} \;=\; & e_1 \left(\frac{p_1'^{\,\mu}}{p_1' \cdot k} - \frac{p_1^{\mu}}{p_1 \cdot k}\right) T_0 \\
& + \left(p_1'^{\,\mu} \frac{p_2' \cdot k}{p_1' \cdot k} - p_2'^{\,\mu} + p_1'^{\,\mu} \frac{p_2 \cdot k}{p_1 \cdot k} - p_2^{\mu}\right) T_5 \\
& + \frac{2\mu\, k \cdot q}{m_1} \left(\frac{p_1'^{\,\mu}}{p_1' \cdot k} - \frac{p_1^{\mu}}{p_1 \cdot k}\right) T_6 \\
& + 1 \leftrightharpoons 2
\end{aligned}
$$

Final amplitude: gravitational case

$$
\begin{aligned}
M^{\mu\nu} &= \left(\frac{p_1'^{\,\mu}\, p_1'^{\,\nu}}{p_1'\cdot k} - \frac{p_1^{\mu}\, p_1^{\nu}}{p_1\cdot k}\right) T_0 \\
&+ \left(p_1'^{\,\mu} p_1'^{\,\nu}\,\frac{p_2'\cdot k}{p_1'\cdot k} + p_1^{\mu}\, p_1^{\nu}\,\frac{p_2\cdot k}{p_1\cdot k} + p_2'^{\,\mu} p_2'^{\,\nu}\,\frac{p_1'\cdot k}{p_2'\cdot k} + p_2^{\mu}\, p_2^{\nu}\,\frac{p_1\cdot k}{p_2\cdot k}\right) T_5 \\
&- \left(p_1'^{\,\mu}\, p_2'^{\,\nu} + p_1'^{\,\mu}\, p_2'^{\,\nu} + p_1^{\mu}\, p_2^{\nu} + p_1^{\nu}\, p_2^{\mu}\right) T_5 \\
&+ 2\mu\, k\cdot q \left\{\frac{1}{m_1}\left(\frac{p_1'^{\,\mu}\, p_1'^{\,\nu}}{p_1'\cdot k} - \frac{p_1^{\mu}\, p_1^{\nu}}{p_1\cdot k}\right) - \frac{1}{m_2}\left(\frac{p_2'^{\,\mu}\, p_2'^{\,\nu}}{p_2'\cdot k} - \frac{p_2^{\mu}\, p_2^{\nu}}{p_2\cdot k}\right)\right\} T_6
\end{aligned}
$$

Non-relativistic reductions..

- For the electromagnetic case(gauge choice $\epsilon_0 = 0$):

$$\vec{\epsilon}\cdot\vec{M} = \frac{\vec{\epsilon}\cdot\vec{q}}{\omega}\,(\frac{e_2}{m_2} - \frac{e_1}{m_1})\,T_0 + (\frac{e_1}{m_1^2} + \frac{e_2}{m_2^2})\,\frac{\epsilon_i\,n_j}{4\,\omega}\,(p^j\,p^i - p'^j\,p'^i)\,T_0$$

$$-\,2\,\mu\,\epsilon_i\,n_j\,(\frac{e_1}{m_1^2} + \frac{e_2}{m_2^2})\,q^i\,q^j\,T_6$$

- Rewriting the second group as

$$\frac{\epsilon_i\,n_j}{4\,\omega}\,(\frac{e_1}{m_1^2} + \frac{e_2}{m_2^2})\,X^{ij}$$

where

$$X^{ij} = \{(p^j\,p^i - p'^j\,p'^i)\,T_0 + 8\,\omega\,\mu\,q^i\,q^j\,T_6\}$$

Non-relativistic reductions: gravitational case

- It is straight forward to carry out the reductions hee too. Adopting the gauge

$$\epsilon_{00} = 0 \qquad \epsilon_{0i} = 0 \qquad \epsilon^i_i = 0$$

the final answer is

$$-\frac{\kappa}{\mu\omega}\,\epsilon_{ij}\,X^{ij}$$

What next?

- The first step is to simplify various terms...

$$p^i\,p^j\,-\,p'^i p'^j \simeq\,-4(p^i\,q^j\,+\,p^j\,q^i\,+\,q^i\,q^j)$$

- Next use

$$p^i\,=\,2\,\mu\,v^i$$

- Finally in the Born approximation

$$T(q)\,=\,\int\,\frac{V(r)}{4\,\pi}\,e^{-i\vec{q}\cdot\vec{r}}$$

- Assume spherically symmetric potentials.

Some useful identities

$$q_i\, T_0\,(q) \;=\; -i\,\int (\frac{\nabla_i\, V}{4\pi})$$

$$\frac{q_i\, q_j}{|\vec{q}|^2}\, T_0' = -\int \frac{e^{-\vec{q}\cdot\vec{r}}}{8\pi}\left(V(r)\,\delta_{ij} + \frac{r_i\, r_j}{r}\, V'(r)\right)$$

$$\omega \;\equiv\; i\frac{d}{dt}$$

Punchlines..

- For electromagnetism:

$$-\frac{i}{4\,\pi\,\omega}\,\langle\,\vec{\epsilon}\cdot\ddot{\vec{d}}\rangle\;-\;\frac{i}{8\pi\omega}\,\epsilon_i\,n_j\,\langle\,\frac{\dddot{Q}_{ij}}{3}\,\rangle$$

- For gravitation, the same quadrupole term!

Octupole: status

- Since in electromagnetic case, one gets dipole in leading, electric quadrupole in the next, and $\frac{v}{c}$ corrections to electric quadrupole(non-universal T_5), it is reasonable to expect octupole radiation also in the case of gravitation.
- Some preliminary results are at hand.
- The LET gives the correct mass dependence

$$\frac{\Delta m \mu}{M}$$

- Several of the terms in the fourth time derivative of O_{ijk} are reproduced, but not all.

Octupole..

- The problem could be because in higher orders one has to be extremely careful in keeping small quantities consistently.
- I plan to carefully reanalyse all the terms that have been neglected to see if that improves the situation.
- I plan to use the feynman graph calculation to guide me in this.
- If I succeed in getting the octupole terms correctly, I would conjecture that all multipoles should be reproduced by the LET's.
- Such an exercise will be important in carrying the LET analysis to higher multipoles.
- One can also use explicit T-matrices to learn more.
- These approaches have the important potential for independent verifications of GR calculations.

Results

- For gravitational radiation I was able to obtain the quadrupole formula.
- My method has the potential to yield the $(\frac{v}{c})^2$ corrections.
- Additionally, two remarkable results on parity violations could be proved. These are stated without their full proof in (NDH, *Experimental tests for some quantum effects in gravitation*, Annals of Phy, Vol 107, p.337, 1977).
- I. The first two terms in parity violation in gravitation induced by the Standard Model vanish!
- II. The leading order fundamental parity violations in gravitation are absent in any theory with a symmetric $g_{\mu\nu}$.
- The stress tensor LET that I had proved with Cho had been proved independently by Deser and Boulware shortly before us using very different techniques.
- These results are complimentary to Duff's demonstration that summing tree diagrams of spin-2 theories yields GR in the low energy limit.

Ongoing work...

- In the case of electromagnetism, the T_5 terms were non-universal, but in the case of gravitation the energy dependence of T-matrices could be universal and hence the T_5 terms too.
- A puzzle in the case of electromagnetic radiation was the apparent absence of magnetic dipole radiation.
- I have resolved this as due to the omission of magnetisation terms in the current.
- In the gravitational case also I should be getting the gravi-magnetic quadrupole part also by including the extensions to Weinberg's LET..

Ongoing..

- An important question is about general space-times.
- Quantum calculations could only be formulated in terms of S-matrices involving asymptotic states.
- Coulomb distorted waves was an improvement, and that could also be thought in terms of non-perturbative infrared aspects.
- For general space-times, all this points to the importance of asymptotic symmetries.
- Recently there has been a lot of interest in the so called super-translations and their connections to soft-graviton theorems(Strominger, Cachazo, Ladha, Sen...).
- I would like to explore the import of my results for those issues.