Ninth Vaidya-Raychaudhuri Endowment
Award Lecture

# Cold Compact Stars : A Laboratory for New Physics

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Indian Association for General Relativity and Gravitation

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# Cold Compact Stars : Laboratory for New Physics

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## 1. INTRODUCTION

I am thankful to the Indian Association for General Relativity and Gravitation (IAGRG) for inviting me to deliver the Vaidya - Raychaudhuri Endowment Award Lecture. I feel greatly honoured. Professor Prahlad Chunilal Vaidya and Professor Amal Kumar Raychaudhuri are distinguished relativists, and they have made significant contributions to the developments of the last six decades in the fields of relativity, cosmology and relativistic astrophysics. They taught and inspired students belonging to almost three generations. The relativity group in the country and IAGRG in particular, owe a lot to them for their inspiring leadership.

In this lecture, I shall review the present status of our understanding of the physics of very compact stars, which have recently been observed (see Table 1) and also outline selectively some of the ongoing research work in these fields. It is known that gravitational interaction is very weak compared to other fundamental interactions, and it is reasonable to neglect it while studying atomic or nuclear processes. However, very strong gravitational fields may occur in highly condensed matter inside these compact stars. These provide a natural laboratory for testing other interactions in extremely dense matter. Although, some laboratory experiments are in progress to study the physics of highly condensed matter (e.g., high energy heavy ion collisions), the results are still preliminary and at best inconclusive. An immediate problem of current interest is to study the nature of constituents and their interactions when the density exceeds the normal nuclear density, $\rho_n \sim 4.6 \times 10^{14} gm/cm^3$. Cold compact stars, where the density reaches a few times the nuclear density, may provide useful information here. A description of these stars cannot be given by the Newtonian theory and one needs the general theory of relativity to study the structure as well the stability of these objects, Mathematical techniques generally used for the study of such compact objects are discussed in the next section.

Till very recently, all stars which are more compact than white dwarfs, but are not massive enough to be black holes, have been given the name neutron stars. The possibility of a star, whose gravitational collapse is prevented by the pressure of the cold degenerate neutron gas was mooted even before neutron was discovered by Chadwick in 1932. Two years later, Baade and Zwicke gave the name neutron star and also proposed that a supernova explosion signals a transition from an ordinary star to a neutron star. In 1939, Tolman, Oppenheimer and Volkoff made pioneering studies on the structure of neutron stars. Most of the subsequent work on neutron stars have made use of the techniques developed by these workers, although a variety of equations of state of matter, depending on the constituents and the density involved, have been studied. In this lecture, we will consider mostly stars which are more compact than the standard neutron stars. Typically, this means stars with a radius $< 10$ km and a mass $\sim 1$ $M_{\odot}$. Some candidates for very compact stars are listed in Table 1.

| Compact Stars | M ($M_\odot$) | Radius (km) |
|---|---|---|
| Her X - 1 | $0.98 \pm 0.12$ | $6.7 \pm 1.2$ |
| 4U 1820 - 30 | 0.8 - 1.8 | $< 10$ |
| 4U 1728 - 34 | $\sim 1.1$ | $< 10$ |
| SAX J 1808.4 - 3658 | (i) 1.435 (SS1)<br>(i) 1.323 (SS2) | (i) 7.07<br>(ii) 6.55 |
| PSR 1937 + 21 | $< 2.4$ | $< 11.5$ |
| RX J 1856 - 37 | $0.9 \pm 0.2$ | $< 6^{-1}_{+2}$ |

Table 1: Some candidates for compact stars.

## 2. TOLMAN-OPPENHEIMER-VOLKOFF EQUATION

The standard method of studying cold compact stars makes use of the equations for hydrostatic equilibrium of a relativistic fluid, known as Tolman, Oppenheimer and Volkoff (TOV) equation. One assumes a static spherically symmetric metric for the interior of the star given by,

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\mu(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{1}$$

The matter inside is assumed to be a perfect fluid with an equation of state

(EOS), $p = p(\rho)$, where $p(r)$ is the proper pressure and $\rho(r)$, the proper energy density at the radial coordinate $r$. Given the EOS, all macroscopic properties of the stars (mass and radius, in particular) can be determined for a given central density, $\rho_c$, by solving the TOV equations:

$$-\frac{dp}{dr} = \frac{1}{r^2}(\rho + p)\Big(2M(r) + pr^3\Big)\left(1 - \frac{2M(r)}{r}\right)^{-1} \quad and \tag{2}$$

$$\frac{dM(r)}{dr} = \frac{1}{2}\rho(r)r^2, \tag{3}$$

where we have made the conventional choice, $8\pi G = c = 1$.

Given the EOS, $p = p(\rho)$, one chooses a value of $\rho_c = \rho$ (r = 0), and starts integrating the equations (2) and (3) from the centre of the star onwards, using the initial condition $M(r = 0) = 0$, and continues till $p(\rho(r))$ drops down to zero at some value of $r = b$, which we interpret as the radius of the star with a central density $\rho_c$. The mass of the star is given by $M = M(r = b)$ and thus, one can study the mass-radius $(M - b)$ curves for various central densities.

The TOV approach has been very useful in the study of neutron stars, particularly where one can model the EOS of the constituents. Considerable results are already available for such stars. However, as the density exceeds twice the nuclear density, the composition as well as the EOS become very uncertain. A number of possible, but exotic compositions have already been considered. These will be discussed in Sections 6 and 7. It is hoped that observational results will soon be available to settle the complex problem of the composition of very compact stars.

However, since the application of TOV equations requires prior knowledge of the EOS, compact objects make the exercise difficult and at most tentative. An alternative approach initiated by Vaidya and Tikekar can provide useful results here. This will be discussed in the next section.

## 3. VAIDYA-TIKEKAR APPROACH

In this method, we choose a suitable geometry for the 3-space and then determine an appropriate equation of state for the matter in the cold compact star. This approach is convenient in cases where the equation of state is not known or uncertain. The method becomes particularly simple when the choice of the 3-geometry permits an exact solution of Einstein's equations. The ansatz of Vaidya and Tikekar [1], for the metric function,

$$e^{2\mu} = \frac{1 + \lambda\, r^2/R^2}{1 - r^2/R^2} \tag{4}$$

has been found to be very useful. Here, $\lambda$ and $R$ are two constant parameters, which measure the spheroidal characters of the $t$ = constant hypersurface, which is spheroidal when embedded in a four dimensional Euclidean space. These parameters will eventually be related to the properties of the constituent matter. Since one of the metric functions is known, using Einstein's equation, one can derive a second-order differential equation for the other metric, $e^{2\nu} = \psi^2$, given by

$$(1 - Z^2)\psi_{zz} + Z\psi_z + (\lambda + 1)\psi = 0\ , \tag{5}$$

where, $Z^2 = \frac{\lambda}{\lambda+1}\left(1 - \frac{r^2}{R^2}\right)$ and $\psi_z$ is the derivative of $\psi$ *w.r.t.* $Z$.

The solution of the equation (5) was obtained by Vaidya and Tikekar for $\lambda = 2$ [1], and by Tikekar [2] for $\lambda = 7$. Maharaj and Leach [3] gave the solution for sets of values of $\lambda$. The general solution, obtained by Mukherjee, Paul and Dadhich [4], is given by

$$e^{\nu} = \psi = A \left\{ \frac{\cos\left[(n+1)\zeta + \gamma\right]}{n+1} - \frac{\cos\left[(n-1)\zeta + \gamma\right]}{n-1} \right\} \tag{6}$$

where $\zeta = \cos^{-1} Z$, and $A$ and $\gamma$ are constants to be determined by matching the solution with the exterior vacuum solution of Schwarzschid at the boundary, $r = b$, i.e.,

$$e^{2\nu(b)} = 1 - \frac{2M}{b} \tag{7}$$

$$e^{2\mu(b)} = \left(1 - \frac{2M}{b}\right)^{-1} \tag{8}$$

The energy-density and pressure are given in this model as

$$\rho = \frac{1}{R^2(1-Z^2)} \left[1 + \frac{2}{(\lambda+1)(1-Z^2)}\right] \tag{9}$$

$$p = -\frac{1}{R^2(1-Z^2)} \left[1 + \frac{2Z\psi_z}{(\lambda+1)\psi}\right] \tag{10}$$

The radius of the star is determined from the condition that the pressure vanishes at the boundary, $r = b$, which gives

$$\frac{\psi'(Z_b)}{\psi(Z_b)} = -\frac{\lambda+1}{2Z_b} \tag{11}$$

The mass of the star is given by,

$$M = \frac{(1+\lambda)b^3/R^2}{2(1+\lambda b^2/R^2)} \,. \tag{12}$$

Each of the two metric functions have two parameters, $\lambda$ and $R$, and $A$ and $\gamma$. Two of these are utilised to match with the exterior Schwarzschild metric, and one is utilised to fix the given input, radius, central density or surface density. We are thus left with one parameter, say $\lambda$, which may be used to characterize the relevant equation of state, which is given implicitly by equations (9) and (10). Thus, the model describes a one-parameter family of EOS for a given $M$ and $b$. It is possible that not all the EOS will be realised in nature.

The model, though very simple, satisfies the physical constraints of a realistic star, if $\lambda \geq \frac{3}{17}$. Following the method of Chandrasekhar, Knutson [5] has also shown that the Vaidya-Tikekar star with $\lambda = 2$ is stable with respect to small radial perturbations. Calculations with the general solution (6) have also confirmed this stability for other values of $\lambda$ [6].

The simplicity of the model have inspired a number of workers [7-9] to use it to study different aspects of compact stars. The model has also been generalised to the case where the star has a charge [10] or anisotropy of some special types [11].

In the context of compact stars, two cases may be mentioned. SAX J1808.4 − 3658 is a compact star in a low mass X-ray binary system and is a candidate for a strange star. Dey, et al. [12], have given an equation of state for strange matter based on a model in which one uses an interquark potential with the following features :

(i) The model has asymptotic freedom.

(ii) It shows confinement at zero baryan number density ($n_B = O$) and deconfinement at high $n_B$.

(iii) The quark mass is a function of density so as to take care of the chiral symmetry restoration, and

(iv) It gives a stable configuration for charge zero, $\beta$-stable strange matter.

The resulting EOS has been approximated by Gondek - Rosinska, et al. [13], as

$$P = a(\rho - \rho_o) \ ,$$

where $a$ and $\rho_o$ are two parameters. Two sets of parameters were considered : (i) $a = 0.463$, $\rho_o = 1.15 \times 10^{15}$ gm/cc, which gives the EOS, SS1, leading to a mass, $M = 1.435\ M_\odot$ and radius $b = 7.07$ km; (ii) the set EOS, SS2 with parameters $a = 0.455$, $\rho_o = 1.33 \times 10^{15}$ gm/cc, which gives $M = 1.323\ M_\odot$ and $b = 6.55$ km. The calculations [6] are done with TOV equations and were repeated in the Vaidya-Tikekar approach, taking $M = 1.435\ M_\odot$ and $b = 7.07$ km as input parameters, and it was found that, for $\lambda = 53.3$, the resulting EOS agrees accurately with the EOS SS1. Similarly, very good agreement was noticed when the parameters of EOS SS2 were considered. The exercise has an important message. It shows that one can get useful information on the EOS of the matter inside the star also from geometric considerations.

## 4. MYSTERY OF MAXIMUM MASS

In relativistic astrophysics the problem of maximum mass of a very compact star is an important issue. This upper bound on mass, in the case of a very compact object, is crucial for making the distinction between a blackhole and a 'normal' compact star. The existence of a maximum mass for a configuration of marginally relativistic fermions can be shown easily by following essentially the arguments given by Landau way back in 1932.

Consider a cold star of radius $R$ containing $N$ fermions so that the number density of fermions, $n \sim N/R^3$; Pauli's exclusion principle gives for the volume occupied per particle $\sim \frac{1}{n}$. We may use Heisenberg's uncertainty principle to estimate $p \sim \hbar n^{1/3}$. The fermi energy of the degenerate fermions can now be estimated :

If the particles are relativistic,

$$E_F \sim pc \sim \hbar n^{1/3} c \sim \frac{\hbar c N^{1/3}}{R} , \tag{13}$$

whereas for non-relativistic particles,

$$E_F \sim p^2 \sim \frac{\hbar^2 N^{2/3}}{R^2} . \tag{14}$$

The gravitational energy per particle is given by,

$$E_G \sim -\frac{GMm_B}{R} = -\frac{GNm_B^2}{R} , \tag{15}$$

where $M = Nm_B, m_B$ being the mass of the baryons. The mass of the star is essentially determined by $m_B$. The total energy per particle (relativistic) is given by,

$$E = E_F + E_G = \frac{\hbar c N^{1/3}}{R} - \frac{GNm_B^2}{R} , \tag{16}$$

which should be minimized to get an equilibrium configuration. We consider the following situations :

(i) If $E$ happens to be negative at the relativistic regime for a given $R$, it will decrease further with decreasing $R$. Thus, there will be no stable configuration.

(ii) If $E$ is initially positive, it will decrease with increasing $R$, but $E_F$ will also decrease. Hence, a stage will come when fermions become non-relativistic and the fermi energy $E_F$ will scale as $\frac{1}{R^2}$. Hence, $E$ will eventually become negative. But, $E$ tends to zero as $R \rightarrow \infty$, showing that $E$ must have a minimum at some value of $R$, which will correspond to an equilibrium state. Also, it follows that $N$ should have a maximum value, $N_{max}$, corresponding to the case where this minimum value of $E$ is zero. Thus,

$$N_{max} \sim \left(\frac{\hbar c}{Gm_B^2}\right)^{3/2} \sim 2 \times 10^{57} , \tag{17}$$

and hence $M_{max} \sim N_{max} \cdot m_B \sim 1.5 M_{\odot}$. We note the following :

(i) $M_{max}$ may be modified by a factor $\sim O(1)$, depending on the EOS.

(ii) $M_{max}$ is determined essentially by values of physical constants, a remarkable result.

(iii) $M_{max}$ will be almost the same for white dwarfs and neutron stars, although degenerate fermions are different in the two cases.

(iv) The argument is based on the assumption that in the high density region, the EOS is given by that of a relativistic free fermi gas. Zeldovich gave a counter example to such a behaviour.

The existence of a maximum mass follows from the relativistic equation of hydrostatic equilibrium and other physical constraints, in particular, the requirement that within the fluid sphere, the velocity of sound be subluminal. The problem will be studied further in the context of Vaidya-Tikekar model in the next section. In particular, the effect of the relevant EOS of the matter inside will be explored in detail.

## 5. MAXIMUM MASS IN VT MODEL

A relativistic star, in general, should satisfy the following conditions:

(i) Both energy density and pressure should be positive in the interior of the star.

(ii) The pressure should vanish at some finite distance from the centre, thus determining the radius of the star.

(iii) Inside the fluid sphere, the velocity of sound $\nu_s$ should satisfy the condition
$0 \leq \nu_s \left(= \sqrt{\frac{dp}{d\rho}}\right) \leq 1.$

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(iv) The fluid sphere should be dynamically stable against small radial perturbations.

We make use of these principles to find the relativistic bounds on the mass of stellar configurations in VT model. From equations (9) and (10), we obtain

$$\frac{dp}{d\rho} = \frac{Z(1-Z^2)^2(\psi_z/\psi)^2 - (1-Z^2)(\psi_z/\psi)}{Z(1-Z^2)(1+\lambda)+4Z} \ . \tag{18}$$

To determine the maximum mass, we consider the particular configuration where $\frac{dp}{d\rho} \leq 1$ everywhere and $\frac{dp}{d\rho} = 1$ at the centre. This yields the condition for maximum mass as

$$\left(\frac{\psi_z}{\psi}\right)_{Z_o} = \frac{1+\lambda}{2\sqrt{\lambda}}\left[\sqrt{\lambda+1} \pm \sqrt{21\lambda+1}\right] \ , \tag{19}$$

where $Z_o^2 = \lambda/(\lambda+1))$. Now for $\lambda > \frac{3}{17}$, positivity of pressure requires that we choose the negative sign in equation (19). The solution of Mukherjee et al., equation (6), can be used to calculate

$$\left(\frac{\psi_z}{\psi}\right) = \frac{(n^2-1)}{\sqrt{(1-Z^2)}}\left[\frac{\sin\left[(n-1)\zeta+\delta\right] - \sin\left[(n+1)\zeta+\delta\right]}{(n+1)\cos\left[(n-1)\zeta+\delta\right] - (n-1)\cos\left[(n+1)\zeta+\delta\right]}\right] \tag{20}$$

where $n^2 = \lambda + 2$.

Combining equation (19) with equation (20) evaluated at the centre, one can determine the limiting value of $\delta$ for a given $\lambda$, and hence the maximal value of $b^2/R^2$. The compactness of the stellar configuration is given by

$$u = \frac{M(b)}{b} = \frac{1+\lambda}{2\left(\lambda+\frac{1}{y}\right)} \ , \tag{21}$$

where $y = b^2/R^2$. It is obvious that the maximum value of $y$ corresponds to the maximum compactness for a given value of $\lambda$.

The method employed can be summarised as follows :

(i) We first specify a value of $\lambda$. A large value of $\lambda \sim 100$ is more suited for describing very compact stars.

(ii) We assume that the maximum mass corresponds to $\left(\frac{dp}{d\rho}\right) = 1$ at r = 0. This determines $y_{max} = \left(\frac{b^2}{R^2}\right)_{max}$ and hence, the maximum compactness.

(iii) There is still one free parameter. It can be specified by giving the value of one of the three quantities : (i) the radius b, (ii) the central density $\rho_o$ or (iii) the surface density $\rho_b$.

This gives us the flexibility to study the maximum mass problem giving any one of these quantities as an input. The mass of the star can be written in terms of the last two quantities as :

$$M = \frac{\sqrt{3}(1+\lambda)^{3/2}y^{3/2}}{2\sqrt{\rho_o}(1+\lambda y)} \tag{22}$$

$$= \frac{(1+\lambda)^{3/2}y^{3/2}(3+\lambda y)^{1/2}}{2\sqrt{\rho_b}(1+\lambda y)^2} \quad . \tag{23}$$

| $\lambda$ | $\left(\frac{b^2}{R^2}\right)_{max}$ | $\left(\frac{M}{b}\right)_{max}$ | $M_{max}/M_{\odot}$ | | |
|---|---|---|---|---|---|
| | | | b = 10 $km$ | b = 8 $km$ | b = 6 $km$ |
| 1 | 0.4618 | 0.3159 | 2.14 | 1.71 | 1.28 |
| 2 | 0.4234 | 0.3438 | 2.33 | 1.86 | 1.39 |
| 3 | 0.3727 | 0.3519 | 2.38 | 1.90 | 1.43 |
| 4 | 0.3297 | 0.3554 | 2.41 | 1.92 | 1.44 |
| 5 | 0.2944 | 0.3573 | 2.42 | 1.93 | 1.45 |
| 7 | 0.2417 | 0.3591 | 2.43 | 1.94 | 1.46 |
| 10 | 0.1898 | 0.3602 | 2.44 | 1.95 | 1.46 |
| 20 | 0.1102 | 0.3611 | 2.44 | 1.95 | 1.46 |
| 50 | 0.0486 | 0.3614 | 2.45 | 1.96 | 1.47 |
| 100 | 0.0252 | 0.3615 | 2.45 | 1.96 | 1.47 |
| 200 | 0.0128 | 0.3615 | 2.45 | 1.96 | 1.47 |

Table 2: Maximum mass ($M_{max}$) of a star for different radii b = 10, 8 and 6 km and for different choices of the parameter $\lambda$ [14].

Sharma, Karmakar and Mukherjee [14] have made a systematic study of the maximum mass problem in the VT model. Some of these results are given in Tables 2 and 3. In Table 4, we have compared our results with results obtained by earlier workers, who made use of the TOV equation. There is a fair amount of agreement, indicating that VT model may have some overlap with realistic EOS for compact stars. This has been shown explicitly by Sharma, et al. [6] in the case of SAX-J. It is expected that when further information about the composition of compact stars become available, the question of the applicability of VT model to realistic compact stars can be settled.

| $\lambda$ | $\rho_b = 5.4 \times 10^{14}$ | | $\rho_b = 10.8 \times 10^{14}$ | | $\rho_b = 4.6 \times 10^{14}$ | | $\rho_b = 5.1 \times 10^{14}$ | |
|---|---|---|---|---|---|---|---|---|
| | $M_{max}$ | $b_{max}$ | $M_{max}$ | $b_{max}$ | $M_{max}$ | $b_{max}$ | $M_{max}$ | $b_{max}$ |
| 1 | 2.61 | 12.19 | 1.84 | 8.62 | 2.83 | 13.21 | 2.68 | 12.55 |
| 2 | 2.78 | 11.93 | 1.96 | 8.44 | 3.01 | 12.93 | 2.86 | 12.28 |
| 3 | 2.78 | 11.66 | 1.96 | 8.24 | 3.01 | 12.63 | 2.86 | 12.00 |
| 4 | 2.76 | 11.47 | 1.95 | 8.11 | 2.99 | 12.43 | 2.84 | 11.80 |
| 5 | 2.74 | 11.33 | 1.94 | 8.01 | 2.97 | 12.28 | 2.82 | 11.66 |
| 7 | 2.71 | 11.15 | 1.92 | 7.88 | 2.94 | 12.08 | 2.79 | 11.48 |
| 10 | 2.68 | 11.00 | 1.90 | 7.78 | 2.91 | 11.92 | 2.76 | 11.22 |
| 20 | 2.64 | 10.79 | 1.87 | 7.63 | 2.86 | 11.70 | 2.72 | 11.11 |
| 50 | 2.61 | 10.66 | 1.85 | 7.54 | 2.83 | 11.55 | 2.68 | 10.97 |
| 100 | 2.60 | 10.61 | 1.84 | 7.50 | 2.82 | 11.50 | 2.67 | 10.92 |
| 200 | 2.59 | 10.59 | 1.83 | 7.49 | 2.81 | 11.47 | 2.67 | 10.90 |

Table 3: Maximum mass ($M_{max}$) in $M_{\odot}$ and corresponding radius $b_{max}$ in km of a star for different choices of the parameter $\lambda$ and surface density in units of $gmcm^{-3}$ [14].

| Reference | Model | $M_{max}/M_{\odot}$ |
|---|---|---|
| Ruffini and Rhoades [15] | Neutron star (Causality principle) | 3.2 |
| Haensel et al. [16] | Neutron star ($dp/d\rho = 1$) | $3.0(5 \times 10^{14} gmcm^{-3}/\rho_b)^{1/2}$ |
| Baldo et al. [17] | BBB2 EOS ($npe\mu$) | 1.92 |
| Kalogera and Baym [18] | Neutron star | 2.2 - 2.9 |
| Witten [19] | Quark star (Bag model) | $2.0(B_0/B)^{1/2}$, $B_0 = 56 MeV fm^{-3}$ |
| Burgio et al. [20] | Quark star (Bag model) | 1.45 - 1.65 |
| Banerjee et al. [21] | Quark star | $1.54(B^{1/4} = 145 MeV)$ |
| Mak and Harko [22] | Strange star | $1.9638/\sqrt{B_{60}}, 2.86$ (charged case) |
| Harko and Cheng [23] | Strange star | 1.83 |
| Cheng and Harko [24] | Strange star | 2.016 |
| Knutsen [5] | Vaidya-Tikekar model [24] | 3.0 |
| Present work [14] | Vaidya-Tikekar model [24] | $3.01(\lambda = 2, \rho_b = 4.6 \times 10^{14} gmcm^{-3})$<br>$2.82(\lambda = 100, \rho_b = 4.6 \times 10^{14} gmcm^{-3})$<br>$2.45(\lambda = 100, b = 10km)$<br>$1.96(\lambda = 100, b = 8km)$<br>$1.47(\lambda = 100, b = 6km)$ |

Table 4: Maximum mass configurations obtained in different models [14].

# 6. COMPACT STARS : COMPOSITION

A typical neutron star has an outer crust (nuclei and electrons), an inner crust (nuclei, neutrons and electrons and muons), an interior part consisting of superfluid protons, neutrons, normal leptons, and a more dense hadronic core. As the star becomes more compact, the core may contain more exotic matter, depending on the density. Various possibilities for the composition of the core are considered below [25].

Hyperon star: Inside the core, the chemical potential of neutrons may exceed the masses (modified by interactions) of the heavier members of baryon octet. One would, therefore, expect to find hyperons $(\Lambda, \Sigma, \Xi)$ in the core. Eventually, even $\Delta$'s may be found in the core of these stars.

Nucleon star: As the core density increases, the reaction $e^- \to k^- + \nu$, becomes energetically favourable. Thus, the fermions $e^-$ get replaced by bosons as $\nu$ escapes. The possibility of this reaction taking place depends on the mass of $K^-$ inside the dense matter of the star. Terrestrial laboratory experiments fortunately provide some useful information here. The kinetic energy spectrum of produced $K$ in Ni–Ni collision was studied by Kaos collaboration at GSI. It was shown that in the attractive nuclear matter at thrice the nuclear density, the mass of $K$ goes down to about 200 Mev (the mass in vacuum is 495 Mev). Note that the chemical potential of electron inside the neutron star matter can very well be close to 200 Mev so that the above reaction can take place. The neutrons can now be converted back to protons [25] and the core of a newly formed neutron star could become iso-spin symmetric, with almost equal number of protons and neutrons. This lowers the energy per baryon of the nuclear matter. The neutron star of this type is called Nucleon star, being very similar to an ordinary nucleus, which contains almost equal number of protons and neutrons. The maximum mass of these stars has been estimated to be about 1.5 - 1.8 times the solar mass.

H-dibaryon star: Dibaryon is an exotic composite of six quarks, doubly strange, with baryon number two, zero spin and zero iso-spin. In the core of moderately dense neutron stars, there is a sizable number of $\Lambda$ hyperons, which could combine to produce H-dibaryons. If the density is about 3 - 6 times the nuclear density, the H-dibaryons can give rise to H-matter, depending on the in-medium properties of H-dibaryons. It has been suggested [26] that in a mild attractive potential of about $-$ 30 Mev, H-dibaryons, which has a vacuum mass of about 2.2 Mev, can form a Bose condensate in the core of a neutron star of mass of about 1.44 times the solar mass. Thus, H-dibaryons could very well be present inside the Hulse-Taylor Pulsar, PSR 1913 - 16. However, if the medium potential is repulsive, say about + 30 Mev, dibaryons cannot be formed unless the mass of the star exceeds 1.6 times the solar mass. However, the H-matter is unstable against a compression and a perturbation can easily convert a dibaryon star into a possible strange star, which will be discussed in the next section.

## 7. A STRANGE STORY

Since 1973, scientists have been studying the possibility that in the extreme pressure prevailing in the core of neutron stars, the constituents, proton, neutron and the hyperons, could melt and undergo a phase transition to produce the quark-gluon plasma. It is still not known at what density this phase transition becomes possible. It is expected that the existence of quark-gluon plasma will be confirmed in near future from the laboratory experiments with heavy ion colliders at CERN, and LHC in particular. There is no definite indication yet from the Lattice QCD simulation results. However, a simple argument can provide a rough estimate of the threshold. Nuclei begin to touch each other at a density,

$$\rho \sim \left(\tfrac{4\pi}{3}b^3\right)^{-1} \sim 0.24 fm^{-3} .$$

For higher densities, the nuclear boundaries should disappear and quarks may become deconfined. Depending on the rotational frequency and mass, a density greater than 3 - 4 times the nuclear density may occur in pulsars and it is likely that deconfinement may set in such pulsars.

The concept of strange matter has become a subject of great interest after the suggestion of Witten [19] in 1984 that it is the true ground state of quantum chromodynamics. The essential idea is as follows. Among nuclei, the nucleus $^{56}Fe$ has the highest B.E. per nucleon, given by $M(^{56}Fe)c^2/56 = 930.4$ Mev. But for strange matter, described by the Bag Model, $E/A = 829$ Mev for the Bag constant $B = 57.5$ Mev/fm and $E/A = 915$ Mev for $B = 85.3$ Mev/fm, both lower than that in Fe. Extensive studies with the Bag model with various values of $m_s$, the mass of the strange quark and the Bag constant $B$ have revealed a window of stability in the $m_s - B$ plane in which the strange quark matter is stable. This implies that one could expect strange matter to provide star-like self-bound systems (strange stars), much more compact than normal neutron stars.

One would expect these self bound stars to consist of a plasma of almost equal populations of $u, d$ and $s$ quarks and a small admixture of electrons.

This presence of strange matter in a star will have startling consequences:

(i) All stellar configurations would be metastable. Also if transition to strange matter is possible (by tunneling !), all neutron stars may eventually become strange stars. Such stars will consist of 3 flavored strange matter covered by a thin crust of hadrons with density lower than the neutron drip density $(4 \times 10^{11} gm/cm^3)$.

(ii) The absolute stability of strange matter may produce a variety of objects, starting from small strange stars with $A \sim 10^2$ to enormous objects $A \sim 10^{57}$, beyond which the configurations become unstable under gravitational collapse.

(iii) If the change over to strange matter takes place within a neutron star, of course, the maximum mass will be the same as that of a typical neutron star.

(iv) However, if a large quantity of quark matter is left in the universe, as a relic of the quark-hadron phase transition in the early universe, it could condense due to gravitational attraction and can even form invisible quark galaxies. The stars of such galaxies would be strange stars. Since these did not evolve out of neutron stars, it is interesting to investigate their maximal mass. Banerjee, et al. [20] following the energy balance arguments, calculated the maximum mass of strange stars. For a typical value of $B^{1/4} \sim 145$ Mev, they obtained the values $R_{max} \doteq 12.11$ km, $M_{max} = 1.54\, M_{\odot}$ and $N_{max} = 1.55 \times 10^{57}$, almost the same as for a neutron star. It was also shown that the maximum mass decreases as $B$ increases.

(v) If strange stars really exist, they will provide useful information about strong interaction of hadrons, in particular, deconfinement and quark-gluon plasma.

## 8. BOSON STARS

Stable soliton-like configurations of a bosonic field bound by its own gravitational field are called Boson Stars (BS). These may be regarded as descendants of geons, proposed by J.A. Wheeler in 1955, which are self-gravitating photonic configurations. The study of BS started in 1968 with the seminal paper by Kaup [27], in which self-bound configurations of a complex scalar field were studied semiclassically, the classical energy-momentum tensor $T\mu\nu$ providing the source for gravitational interaction. This was followed by Ruffini and Bonazzola [28], who considered the quantization of a real scalar field and constructed the ground state of N particles. The vacuum expectation values (VEV) of the field operators here lead to the same energy-momentum tensor (and hence, the same field equations) as are obtained with classical complex scalar fields. The gravitational field, $g_{\mu\nu}$, was, however, treated as a classical field. Considerable work has since been done on BS, considering a wide variety of scalar fields and their possible interactions. The interactions considered are of the following types :

(i) The fields considered are massive and complex but have no other self-interactions.

(ii) Fields may have a self-interaction described by a potential, V($\phi$) or V($\phi\phi^*$), depending on whether $\phi$ is a real or a complex scalar field.

(iii) Minimal coupling to gauge fields, where the scalar field carries a charge, e.g., electric charge or hypercharge.

(iv) Non-minimal coupling, as in scalar-tensor theories, or self-interacting dilaton fields.

The general features of BS are outlined below :

(i) BS are essentially macroscopic quantum states. If we consider massive scalar fields without any self-interaction, these states are protected against a gravitational collapse by Heisenberg's uncertainty principle. These states are usually called mini-boson stars.

(ii) It it easy to estimate the order of the mass of a mini-boson star. If the state has a characteristic size R, the uncertainty principle suggests that the momentum $p \sim \frac{1}{R}$. For a moderately relativistic boson, we expect $p \sim m$ (the boson mass), so that $R \sim \frac{1}{m}$. Now, for hydrostatic equilibrium, the total mass of the boson star $M \sim \frac{R}{G}$, which we can rewrite as $M \sim \frac{1}{Gm} = \frac{Mp^2}{m}$ . This also shows that one can get boson stars of different masses, depending on the mass of the scalar field.

(iii) The above estimate of the BS mass is much smaller than the Chandrasekhar mass for stars with marginally relativistic fermions, e.g., $M_f \sim \frac{M_P^2}{m^2}$

(iv) The situation, however, changes if one considers a self-interaction. Colpi, Shapiro and Wasserman [29] considered a complex scalar field with $\lambda(\phi^*\phi)^2$ interaction and showed that the resulting configuration differs considerably from the non-interacting case even for a very small $\lambda$. The maximum mass in this case could be large and comparable with Chandrasekhar mass for fermions. This makes the study of BS relevant in Astrophysics. If BS really exists, it may account for a part of the dark matter of the universe. Thus, BS may play a very important role, acting as a laboratory for testing various ideas for dark matter, as well as for models of scalar fields and their interactions.

(v) The BS considered above are non-perturbative solutions of Einstein-Klein-Gordon equations, as is obvious from the relation $M \propto \frac{1}{G}$. Thus, there is no flat space-time limit for these objects.

(vi) The BS is not a static object. The scalar field has the form of a standing wave, $\Phi(\overrightarrow{r}, t) = e^{iwt}\phi(\overrightarrow{r})$. However, the metric and the tensor $T\mu\nu$ of the scalar field are time independent.

## 9. IN SEARCH OF A SCALAR FIELD

Although particle theories make use of a large number of scalar fields, none of these has yet been observed. There are of course, pseudo scalar mesons. These are described by complex scalar fields. The Klein-Gordon equation for a complex scalar field has a global U(1) symmetry and this leads to a conserved quantity. For charged pseudo scalar particles, say, $\Pi^+$ and $\Pi^-$, the conserved quantity is the charge

$$Q = e\,(N^+ - N^-)\ ,$$

whereas for neutral particles, e.g., $K^o$ and $\overline{K}^o$, the conserved charge is the strangeness S. These particles are unstable and it is not clear if these can form a stable bound state by gravitational binding.

The most likely scalar field is the Higgs particle. In Salam-Weinberg model, Higgs boson doublet $(\Phi^+, \Phi^o)$ and their antiparticles $(\Phi^-, \overline{\Phi}^o)$ are necessary for generating masses for the gauge vector mesons $W^\pm, Z^o$. After the symmetry breaking, only one real scalar particle $h = \frac{1}{\sqrt{2}}(\Phi^o + \overline{\Phi}^o)$ remains free and its mass is expected to be of the order of 1 $Tev/c^2$. Supersymmetric extensions would imply more Higgs fields, $H^o, A^o$ and a charged doublet $H^\pm$ in the mass range of 100 $Gev/c^2$ – 1000 $Gev/c^2$. Experiments planned in near future should be able to confirm if these particles exist. The possibility of the formation of stable Boson stars with the Higgs field is a problem of considerable interest. In SW model, if $h$ is much heavier than gauge bosons, we expect possible decay modes $h \to W^+ + W^-, h \to Z^o + Z^o$ to exist. In the BS, these decay channels will be in partial equilibrium with the inverse process, e.g., $Z^o + Z^o \to h$, $Z^o + Z^o \to Z^o + h + \gamma$, etc., in which the participating particles borrow from the gravitational binding energy. This is similar to the neutron star, where the $\beta$-decay and the inverse $\beta$-decay processes maintain the equilibrium of the macroscopic body. Considerable work has also been done on dilaton star.

## 10. CONCLUDING REMARKS

The possible existence of highly compact stars has been a subject of study for years. An interesting result in this connection was obtained by Iyer, Vishveshwara and Dhurandhar [30], who considered the available EOS and concluded that an object with a radius $b < 3M$, where $M$ is the mass, could very well exist. They called these objects "ultra-compact". Recent detection of compact dead stars is consistent with this prediction. This also opens up the possibility of studying matter at densities much higher than the nuclear density and also provides constraints on models of particle interactions. Presently, we do not have any reliable information about the EOS of these compact objects. The EOS derived from known nuclear physics and other terrestrial experiments (e.g., heavy ion scattering) cannot possibly be extrapolated upto the densities relevant for such objects ($\rho > 2\rho_n$). Vaidya-Tikekar model can play a very useful role here. The model has already been used with advantage to study a strange star [6]. Recently, Sharma, Karmakar and Mukherjee [14] have used the model to make an extensive study of various aspects of the problem of maximum mass for compact VT stars.

It should be pointed out that VT model, in general, may not describe a realistic star. If it does, it is a welcome coincidence. The coincidence is more likely if the EOS is almost linear. This has already been confirmed in the case of SAX-J [6]. The results on maximum mass are also in general agreement with those calculated with the known EOS, as are shown in Table 4. Thus, there seems to be some overlap of the VT model with realistic EOS of compact stars. Whether this overlap survives when further information on the EOS of very compact stars become available is an issue of great interest. Meanwhile the VT model may be considered as a toy model for a class of cold compact stars. The simple analytic solution [6] is an added advantage of this model. It has already been used [31] in a perturbative calculation of the evolution of a star after a supernova explosion, where the solution [6] is used to describe the final state of the star.

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