Chipping in to detect and locate the elusive gravitational wave sources: Looking back and looking forward

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I. DEDICATION

I am honored and humbled to receive this award associated with the two doyens of general relativity (GR), Vaidya and Raychaudhuri, in the centenary year of GR. They were an inspiration to all of us exemplifying that impossible is nothing and circumstance a state one can go beyond. Who in addition to leaving their mark on the areas in GR they excelled in, were great teachers and who moulded generations by their academic integrity, values and simplicity. It is thus appropriate that I dedicate this talk to all my teachers in school, college and university who shaped me into what I became. In particular my Ph.D guide Arvind Kumar and MSc teacher A.A. Rangwala. Arvind Kumar, in whom I observed that if anything is worth doing it is worth doing well and importantly in time and would always require making a sacrifice in giving up other things that you were involved in. On this occassion, I would like to dedicate it to my parents and also extend this dedication to all the parents of India who go far beyond themselves and their means to give their children the best education they can provide them. Next on the list is Vishu who offered me my first and only post-doc and was a collaborator for over a decade. On a lighter vein, I also need to acknowledge the mysterious bug that got Sanjeev Dhurandhar due to which Sanjeev had to move out of Bangalore, inadvertantly creating a potential phase space for me at RRI! I assure you that I had no part in it!! But who knows how it would have been for me..I can only say.. Que sera sera..Via Vishu I could meet the who's who in GR when they visited RRI and become part of the wonderful family of international relativists. We also worked together with Jayant Narlikar, Ajit Kembhavi, A.R. Prasanna, P.C. Vaidya, A.K. Raychaudhuri, Naresh Dadhich, on the UGC instructional conferences on GR, ICGC with the view to bring a mini ISGRG meeting to India every four years since not many relativists from India can even today manage to travel to the ISGRG meetings abroad. Finally, the REAP program at BASE in the Jawaharlal Nehru Planetarium, a physics study circle over the weekends to prepare undergraduate students for a research career in physics. This talk is warmly dedicated to all my collaborators on gravitational waves (GW). Beginning with Thibault Damour, a mentor over the years, who offered me, an outsider to GW, a sabbatical to work on GW; Luc Blanchet with whom I have a wonderful collaboration for more than two decades; Sathyaprakash with whom I and my students regularly worked on GW Data analysis applications; Guillaume Faye, Cliff Will, Gilles Esposito Farese; all my students and postdocs, in particular, A. Gopakumar, Moh'd Qusailah, K.G. Arun, Siddhartha Sinha, Chandra Kant Mishra, P. Ajith, Pranesh Sundarrajan, Ryuichi Fujita. My other mentors Cliff Will, Bernard Schutz, Kip Thorne and Stan Whitcomb..To RRI where in spite of my strong differences on how RRI should or should not be run, I could meander like a river to the ocean I chose to end in.. To Manjunath for secretarial and logistical support in all I was involved in. Finally and most importantly to my wife Suman and daughter Shashwati for putting up with the many absences from home over the years without which I could never have progressed in my endeavour to chip in to detect and locate the elusive GW sources.

II. PROLOG

Einstein's theory of gravity, the General theory of Relativity was given in 1915. It is universally considered as the epitome of mathematical elegance, conceptual depth and importantly observational success. 2015 is the centenary of General Relativity. In the paper exploring physical implications of GR in 1916, Einstein proposed the existence of GW as one of its **important** consequence. However, unlike for electromagnetism (EM), there has been no experimental confirmation à la Hertz. The reason is fundamental and connected to two basic differences between EM and GR: The weakness of the gravitational interaction relative to EM (10^{-39}) and the massless spin two nature of GR compared

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to massless spin one nature of EM that forbids dipole radiation in GR. This implies low efficiency for conversion of mechanical energy to gravitational radiation except in strong fields and at relativistic velocities. And feeble effects of GW on any potential detector. Likely sources are signals produced by astrophysical systems where there are potentially huge masses accelerating very strongly. These include bursts from supernova, chirps from coalescing neutron star and black hole binaries, GW from isolated neutron stars and optimistically stochastic murmurs from the big bang.

A. Chandrasekhar as inspiration and role model

When I started my Ph.D work with Arvind Kumar at the Bombay University on the Dirac equation in Kerr spacetime, we were not aware that the separability of the Dirac equation was a hard open problem. Fortunately, by the time we finished this work, Chandra had succeeded in separating it using the Newman Penrose form of the Dirac equation. This work not only made my thesis possible but was an inspiration. The icing on the cake was being quoted in Chandra's black hole opus for our brief paper on absence of Dirac superradiance in Kerr BH. It was a pointer to modern techniques I needed to pick up beyond my standard old style education in GR and became the foundation for work on perturbations in GR with Vishu at RRI during 1980-90. Chandra's work on radial perturbations was an important ingredient for our work on existence of ultra compact objects ($R < 3M$) in GR. Meeting Chandra at RRI two or three times was an experience. I felt that even modulo my limitations, Chandra's mathematical approach to physical problems suited my mental makeup and strengths. Chandra became a role model. Chandra's strategy of moving on from one area to a different one every decade made a deep impression on me. After a decade at RRI, working on problems of GR with some potential astrophysical applications or mathematical issues related to separability of wave equations, I felt I should re-assess and re-orient myself afresh. Based on exposure to a *Gravitation in astrophysics* school in Cargese (1986), GR11 in Stockholm (1986) and ICGC-I in Goa (1987), I felt that early universe cosmology, Ashtekar variables or gravitational waves were possible areas to move into. I applied in 1989 to seek opportunities in the above three areas. The one that worked, strangely, also had a deep Chandra connection (See Sec III C). It represents phase I of this lecture, the period 1989-2009, when I could chip in to help detect the elusive gravitational wave sources.

B. The sabbatical that changed my focus

During 1989-1990 I took my sabbatical with Thibault Damour who was moving then from DARC, Meudon to IHES, Bures-Sur-Yvette. In that sense, I was Thibault's first 'postdoc' at IHES. This proved to be a new phase in my scientific life; one that kept me occupied close to two decades. I began working on problems related to gravitational radiation; the Multipolar Post Minkowskian (MPM) formalism of Luc Blanchet and Thibault Damour. I still remember that when I reached my office in Meudon in September 1989, Thibault was away. But on my desk was the material I had to start on. A copy of Luc Blanchet's Thesis and their bright yellow DARC preprint on `PN generation of GW`. When I met him later that week, the brief was clear. Extend the multipole expansion method using symmetric trace-free (STF) tensors that was implemented for massless vector fields to massless tensor fields and obtain multipole expansion of linearized gravity i.e. `Linearized gravity using STF methods`. Over the next three months I picked up requisite aspects of STF methods and before Christmas had the first cut results. Over the next few months we cleaned it up and noted that corresponding results using tensor spherical harmonics by Campbell, Macek and Morgan referred to in Thorne's classic Reviews of Modern Physics on `Multipole expansions of gravitational radiation` had incorrect coefficients. This brought home to me the computational and algorithmic advantage of STF methods. Thibault then explained how we now had the tools to extend the 1PN computation of mass moments to 1PN current moments. This led to the second project on `1PN current quadrupole with compact support.` I was unaware of technical issues when I worked on this problem about extensions using 'distributional' solutions required to deal with higher non-linearities. Without deep insight, I learnt to work with such objects and took my baby steps into this area working on it when I continued my sabbatical after IHES at MPI (Garching), ICTP, Trieste and Queen Mary, London. On my return to India, I did not have any deep problem in this area to work on. But to keep in touch with the area, I began to generalize it to the 1PN current octupole and collaborate remotely with Thibault. Let me rewind now to history of GW, the territory I was moving into.

III. BRIEF HISTORY OF GRAVITATIONAL WAVES

In 1918, Einstein calculated the flux of energy far from the source, the famous quadrupole formula. He discussed radiation reaction, radiation damping and distinguished between energy carrying waves and non-energy carrying wave-like coordinate artefacts. In 1922, Eddington corrected a factor of 2 in Einstein's work and pointed out the inapplicability of Einstein's derivation for self gravitating systems. He realized that issues of gauge effects were subtle and famously exclaimed `GW propagate at speed of thought!` For long this subtlety continued to vex. For instance, in 1936, Einstein himself wrote to Born. "Together with a young collaborator (Rosen), I arrived at the interesting result that `GW do not exist`, ...". Fortunately, he was convinced about his erroneous interpretation in time and quietly retracted it by the time paper was published. Landau Lifshiftz (1941) and Fock (1955) extended the quadrupole formula to weakly self-gravitating systems. These constitute two different approaches to GW generation today: DIRE and MPM-PN. Where does the complication in GR come from? For self-gravitating, systems orders in velocity are related to orders in non-linearity. By the virial theorem, $\Phi = GM/R$ is of the same order as v^2. Thus reaction terms of order $(v/c)^5$ in a linear theory can be accompanied by terms $(v/c)^3 \, \Phi/c^2$, $(v/c)^1 \, (\Phi/c^2)^2$ for self-gravitating bodies in non-linear GR. Thus, higher order PN calculation requires dealing dealing with higher order non-linearities going beyond linearized gravity.

A. Chapel Hill - 1957 GR1

[1] In 1957 a conference on *The role of gravitation in physics* was held at Chapel Hill, supported by the US Air Force (in a program under Joshua Goldberg). It was organized by Bryce and Cecile Dewitt and is today considered as GR-1. In this meeting among other things the GW problem was solved by Pirani and the quest for GW detection born. Pirani focused the analysis on the `effect` of GW rather than their `generation` and showed that the effect of GW was determined by the equation of geodesic deviation. In the presence of GW a set of freely falling particles would experience genuine motion relative to each other. Thus GW were real. In the discussion following his presentation of which there is a record, Bondi asked:

`Can you connect two masses with a dashpot thus absorbing energy from the wave.` Pirani replied: `I have not put an absorption term but put in a spring. You can invent a system with such a term quickly.`

Bondi and Feynman (who spoke later in this meeting) are credited with the sticky bead argument today in the literature. Thus GW are physical and can in principle heat a suitably contrived mechanical system! The tidal motion of particles due to GW generates heat by the rubbing of particles against the rod! GW can be made to do all sorts of strange things by choice of coordinates, but the way they interact with matter must always make sense physically. Soon after, in 1962, using a mathematically precise discussion of asymptotics in GR, Bondi, Metzner and Sachs proved rigorously that GW transfer energy!

B. The tragic pioneer - Joe Weber and the Bar detector

Weber and Wheeler were also at the Chapel Hill meeting. In 1960 came the Weber bar idea and in 1969 Weber's first claim of GW detection. He claimed to have found coincident excitations of bars at Aragone National Lab and University of Maryland coming from galactic centre. However, as pointed out by Field, Rees and Sciama the observations implied such a huge amount of conversion of stars into GW that it would lead to weakening of the binding and consequent expansion of the galaxy. Starting 1972 other groups (Braginsky, Tyson, Drever, Garwin and Levine,..) tried but failed to replicate Weber's results. Other refutation of Weber's claims were given by Schutz and Grischuk. For more details of this story, refer to the fascinating book *Gravity's Shadow* by Harry Collins.

C. Chandrasekhar and Gravitational Waves

In the 1960's, Chandrasekhar addressed the radiation reaction problem. How does emission of GW affect the emitting system when it is self-gravitating? Chandra was the first to show conceptually that radiation reaction problem could be solved for continuous systems. This gave astrophysicists confidence that GR was physically reasonable and well behaved. Energy and angular momentum radiated as GW was correctly balanced by the loss of mechanical energy and and angular momentum by the source. By brute force, insight and attention to detail, Chandra first achieved what many relativists had tried for decades. But there was a problem. In the gauge he used, some terms at 2PN were

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[1] Credit: Subsections III A,B,G are partially based on a talk by Peter Saulson on this topic

divergent. These divergences cast doubt on the validity of Chandra's treatment for more mathematically demanding relativists. Chandra (and Thorne) did not find the infinities worrying because they felt they had a physicists intuition for the correctness of the method used and results obtained. Chandra was unhappy about the criticism regarding divergent terms since it prevented him from being given *adequate* credit for significance of his PN work. No wonder, that this is the only body of his work *not immortalized* by a book unlike all his other research endeavours!!

D. Discovery of 1913+16, Mark II approximation methods

In 1974 came Hulse and Taylor's discovery of the Binary Pulsar 1913+16. The prospects of testing PN theory against Hulse-Taylor system once again revived more critical questions (Kerlick, Ehlers, Havas..) regarding existing treatment of GW (Chandrasekhar, Thorne..). The high quality binary pulsar data forced a revisit to approximation methods in GR to remedy the mathematical shortcomings in the existing approaches. Insights of a newer generation used to techniques in field theory to deal with 'divergences' probably helped. Eg Damour critically looked at the problem and realised the need to carefully deal with ultra-violet divergences arising from the use of delta functions to model point particles in a non-linear thory. He proposed iteration algorithms including Riesz regularisation to deal with such divergences and iterated Einstein's equations to sufficient order of non-linearity to obtain EOM of compact binaries including v^5/c^5 terms (1983). (Damour and Deruelle, Walker and Will, Schäfer, Grischuk and Kopeijkin, Thorne, Schutz..)

E. Towards Direct Detection of GW

Radio (timing) observations of binary pulsars establish the `reality` of GW, validity of GR in strong fields. The evidence is `excellent` but the evidence is `indirect`. Can detectors be built to attempt a `direct` detection of GW? To address this, we recall that the strength of GW is quantified by the strain h it induces in a system: $h = \frac{\Delta L}{L}$. According to GR, $h(t) = \frac{1}{R}\frac{2G}{c^4}\ddot{I}(t)$, where I denotes the quadrupole moment of the source. Thus, a neutron star neutron star (NS-NS) system in the Virgo cluster of galaxies produces a strain of $h \simeq 10^{-21}$ leading in km scale interferometers to a 10^{-18}m displacement. The miniscule strain and associated tiny displacement of one thousandth of a fermi must be measured to detect GW.

F. Beyond Detection of GW

The direct detection of GW is the first mandate of Laser Interferometric GW detectors. The promised and real excitement is because it would be a new observational window into the dark universe, progressing gradually into a tool for astrophysics and cosmology and eventually an experimental probe for fundamental physics. This is because; (i) the direct detection of GW probes strong field regime of gravitation, (ii) GW observations are privy to aspects not accessible to the EM astronomy and thus can uncover *new* aspects of the physics, (iii) GW must occur in any relativistic theory of gravity but properties of GW in different theories can be diferent.

G. Laser Interferometers for GW detection

In 1963 Gerstenstein and Pustovit proposed the use of laser interferometry to detect GW. In 1964 at MIT Rai Weiss while teaching GR asked, What is measurable in GR, and he found the answer in Pirani, who had not put spring or dashpot but said:

`It is assumed that an observer by use of light signals or otherwise determines the coordinates of a neighbouring particle in his local cartesian coordinate system.`

Zacharias Lab at MIT was deep into the new field of lasers and Rai realized lasers could do the job. In Rai Weiss's instruction book of 1972, as a homework problem, one can say, Rai envisioned LIGO. Interferometers interestingly also figure in Weber's notebook in 1978. Weber worked with Forward on a small interferometer. Soon groups at Munich, Glasgow, Garching, Australia moved towards interferometers for GW detection. At NSF, Richard Isaacson supported engineering design studies and in 1979 NSF funded Caltech and MIT for laser interferometer R&D. At Caltech in 1979 Thorne set up an experimental group with Drever. Drever brought to GW detection the techniques like Fabry Perot, power and signal recycling. Pioneers like Whitcomb (1980-85, 1991-97) and Zucker (1983-94) worked on the 40 m Caltech prototype. Other well-known researchers included Hough, Norna Robertson, Billing, Ruediger, Maischberger, Schilling, Lise Schnupp, Winkler, Brillet, Meers, Giazotto and Blair. In 1985 Winkler et al made a proposal for large GW antenna in Germany. 1989 was the year of big proposals: 3 km Virgo (May), 3 km GEO (Sept), 2x4 km LIGO (Dec). However, Germany re-unified in 1990 and the German 3 km took a backseat. Though many well-known astronomers strongly lobbied against LIGO funding in 1990, National Science Board (NSB) approved the LIGO Construction proposal. LIGO was funded in 1991.

In 1993 the Nobel Prize was awarded to Hulse and Taylor `for discovery of a new type of pulsar, a discovery that has opened up new possibilities for study of gravitation.` Invited to the Nobel cermony were Jocelyn Bell, Thibault Damour and Cliff Will. In the same year, the NSF Review panel endorsed the technical status of LIGO and in 1994 began the site construction at Hanford and Livingston. LIGO inauguration took place in 1999 and in 2003 Virgo joined the search for GW. LIGO design sensitivity was reached in 2005 and in 2007 joint data analysis by LIGO and Virgo was ratified. GW detection related proposals in India happened too. In September 1990 the proposal on: *Interferometric GW Detector Development work and experiment* by S.V. Dhurandhar, N. Dadhich, J.V. Narlikar and S. Tandon of IUCAA and P.K. Gupta, A.S. Raja Rao and D. Bhawalkar of RRCAT (1.25cr INR 21 staff; 100m; 13cr INR, 51 staff) was considered but not funded. December 1995 saw the proposal on *Design of vacuum system of AIGO 500* by A.S. Raja Rao of CAT. Political events of 1998 once again influenced matters in the negative.

IV. PROTOTYPE SOURCES - INSPIRALLING COMPACT BINARIES

The late inspiral and merger epochs of compact binaries of neutron stars or black holes provide us possible strong sources of GW for terrestrial Laser Interferometer GW detectors like LIGO and Virgo in the 'high' frequency range 10 Hz - 10 kHz. We have guaranteed sources for the GW detectors if there are *enough* of them. The waveform is a chirp a signal with amplitude and frequency increasing with time. However, GW are WEAK SIGNALS buried in NOISE of detector and require matched filtering (MF) both for their detection or extraction and parameter estimation or characterisation. Success of MF requires an accurate model of signal using a theory of gravity like general relativity. This favours sources like coalescing compact binaries (CCB) (made of a pair of neutron stars or black holes or a neutron star, black hole system) over unmodelled sources like supernovae or GRB and lead to tremendous progress in the theoretical 2-body problem in GR.

A. Gravitational Machines for Interstellar communication

In 1963 Freeman Dyson wrote a paper entitled: "Possible use of gravitational energy by advanced civilizations by creation of binary neutron star systems to accelerate spacecraft to enormous speeds!" To quote:
`'If a close binary system could ever be formed from a pair of neutron stars, these systems would emit sufficient quantities of gravitational radiation (on account of the intense fields produced at short range by such highly condensed bodies) to cause the system to decay on a relatively short timescale until its components plunged into each other in a final immensely strong burst of GW at frequency suitable for detection by Weber's instrument to distance of order 100 Mpc (100 million galaxies)..would be worthwhile to maintain a watch for events of this kind..` '[2]
Indeed this is very *prescient*. Recall that pulsars were only discovered by Jocelyn Bell and Tony Hewish only in 1967 and binary pulsars even later in 1974! It is interesting too that Dyson suggested to Weber that asymmetric supernova collapse could be possible sources for his Weber bars.

B. The last three minutes..

When LIGO was funded in early nineties, and efforts to construct accurate CCB waveforms started, it was soon realised that far higher order PN accurate waveforms would be needed to approximate GW in the final stages of inspiral and merger than existed in the context of the binary pulsar work. Numerical Relativity was far from mature and a Grand Challenge Program was started towards this goal. At this juncture, physical insights were essential to

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[2] It is as if G/c^5 in the general formula for far zone flux becomes c^5/G for strongly gravitating relativistic sources wrote David Blair recalling a discussion with Weber on this in the volume *The detection of GW* that David edited.

simplify the goals and achieve the required waveforms in the time frame set by the need to be ready for initial LIGO data analysis during the science runs.. They comprise (Cutler et al 1993) (i) garden variety inspiraling compact binaries (ICB) would have radiated away their eccentricity and be moving in quasi-circular orbits during the late inspiral, (ii) since matched filtering is sensitive to the phase it is more important to first control higher order phasing than higher order amplitudes - One could use the Newtonian amplitude but needed the best available phasing; an approximation referred to in the literature as the restricted waveform, (iii) the inspiral can be treated in the adiabatic approximation as sequences of circular orbits. This allows one to treat separately the radiation reaction effects and the conservative effects. This permits one to go to higher PN orders in the inspiral without getting technically bogged down in controlling the much more difficult higher order conservative PN terms., (iv) for compact objects the effects of finite size and quadrupole distortion induced by tidal interactions are of order 5PN. Hence, neutron stars and black holes can be modelled as point particles represented by Dirac δ-functions.

Thus modelling the ICB waveforms for inspiral involves three tasks: (i) Motion: Given a binary system, iterate Einstein's equations (EE) to discuss conservative motion of the system. Compute conserved energy E (and also angular momentum J^i for binaries in eccentric orbits), (ii) Generation: Given the motion of the binary system on a fixed orbit, iterate EE to compute multipoles of the gravitational field and hence the far zone flux of energy (and angular momentum for the eccentric case) carried by GW. Compute $\mathcal{L}$ (and $\mathcal{J}$) and, (iii) Radiation Reaction: Given the conserved energy and radiated flux of energy (and angular momentum in eccentric case), *assume* the balance equations to compute the effect of radiation on the orbit. Compute evolution of the GW frequency $F(t)$ and phase $\phi(t)$ (and separation $r(t)$).

C. MPM-PN formalism

Successful wave-generation formalisms are a cocktail of post-Minkowskian (PM) methods [expansions in G - non-linearity expansions], post-Newtonian (PN) methods [expansions in $1/c$], multipole (M) expansions [expansions in irreducible representations of the rotation group], far zone expansion [expansion in $1/R$] and perturbations around curved backgrounds. There are two independent aspects addressing two different problems. (i) The general method (MPM expansion) applicable to extended or fluid sources with compact support, based on the mixed PM and multipole expansion matched to some PN (slowly moving, weakly gravitating, small-retardation) source. Infra-red divergences arising from the retardation expansion dealt by analytic continuation (ii) The particular application to describe inspiralling compact binaries (ICB) by use of point particle models. Self-field regularisation to deal with ultra-violet divergences arising from use of delta functions to model point particles. These include Riesz, Hadamard partie finie, dimensional regularisation

1. *2PN Phasing for quasi-circular inspiral*

Following funding of LIGO (USA), issues related to templates for GW detection were intensely studied by Kip Thorne's group at Caltech. An International meeting (1994) was convened by Kip to brainstorm this issue and highlight need to address this problem in time. Luc Blanchet and I were participants at this meeting. Luc expressed the view that MPM formalism could be effectively generalised to do this and soon demonstrated the 2PN generation of GW. Soon after I was visiting Thibault at IHES and this led to a collaboration between the three of us and we completed the 2PN phasing for ICB (1995). The new insight it required related to the treatment of the cubic non-linearities. The availibility of the 2PN EOM from the binary pulsar work facilitated the computation of 2PN phasing of ICB by two independent methods: the MPM-PN method used by us and independently, results obtained by Will and Wiseman in the USA using the Direct Integration of Relaxed Einstein equations (DIRE) formalism. All computations at this order could be done by hand by this American French Indian collaboration. The power of email, internet and remote collaboration crucial for complex and human resource intensive projects like LIGO and Virgo was coming into play.

2. *3PN Phasing for quasi-circular inspiral*

Though 2PN templates seemed adequate for binary neutron stars, it was clear that for binary black holes the 3PN approximation would be necessary. However, the computation involved too many terms to be done by hand. Needed algebraic computing tools which we had not used before and we needed to be sure Maple, Mathematica and MathTensor produced the same results at 2PN as obtained by us earlier. We used our existing 2PN results to understand nuances of Mathematica in the beginning and starting 1996 began looking at both the 3PN EOM (Blanchet, Faye, Jaranowski and Schäfer) and 3PN wave generation (Blanchet, Iyer, Joguet). Control of this next order turned out to be more formidable due to limitation of earlier regularisation methods for the self-field using Hadamard regularisation. The 3PN EOM (Damour, Jaranowski, Schäfer; Blanchet, Faye;

Damour, Esposito-Farese 2004, Futamase and Itoh) and 3PN wave generation (Blanchet, Iyer, Joguet, Damour, Esposito-Farese) and 3PN GW polarizations (Blanchet, Iyer, Arun, Qusailah, Faye, Sinha, Mishra) were technically more involved due to ambiguities in Hadamard regularisation. Blanchet and Iyer provided the Hadamard regularisation of 3PN generation (2005). Only after almost a decade of struggle and by the use of the gauge invariant *dimensional regularisation* was the problem finally resolved and completed. It brought together many of us (Blanchet, Damour, Esposito-Farese, Jaranowski, Schafer and Iyer) in different combinations; many of us who had worked together on different aspects of gravitational radiation problems earlier. After almost a decade the basic inputs to construct 3PN accurate templates for GWDA in the adiabatic approximation i.e. the 3PN conserved energy $E_3(x)$ and the 3.5PN energy flux $\mathcal{L}_{3.5}(x)$ were computed. In terms of $x \equiv (GM\omega/c^3)^{2/3}$, where ω is the orbital frequency and ν the dimensionless mass ratio they are given by

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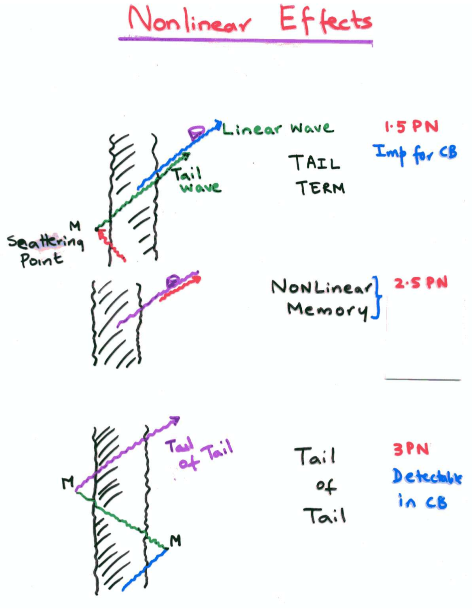
\begin{aligned}
E_3(x) &= -\frac{1}{2}\nu x\left[1-\left(\frac{3}{4}+\frac{1}{12}\nu\right)x-\left(\frac{27}{8}-\frac{19}{8}\nu+\frac{1}{24}\nu^2\right)x^2\right.\\
&\quad\left.-\left\{\frac{675}{64}-\left(\frac{34445}{576}-\frac{205}{96}\pi^2\right)\nu+\frac{155}{96}\nu^2+\frac{35}{5184}\nu^3\right\}x^3\right],\\
\mathcal{L}_{3.5}(x) &= \frac{32c^5}{5G}x^5\nu^2\left\{1+\left(-\frac{1247}{336}-\frac{35}{12}\nu\right)x+4\pi x^{3/2}\right.\\
&+\left(-\frac{44711}{9072}+\frac{9271}{504}\nu+\frac{65}{18}\nu^2\right)x^2+\left(-\frac{8191}{672}-\frac{535}{24}\nu\right)\pi x^{5/2}\\
&+\left(\frac{6643739519}{69854400}+\frac{16\pi^2}{3}-\frac{1712}{105}C-\frac{856}{105}\ln(16x)\right.\\
&+\left.\left[\frac{41\pi^2}{48}-\frac{134543}{7776}\right]\nu-\frac{94403}{3024}\nu^2-\frac{775}{324}\nu^3\right)x^3\\
&+\left.\left(-\frac{16285}{504}+\frac{176419}{1512}\nu+\frac{19897}{378}\nu^2\right)\pi x^{7/2}+\mathcal{O}(x^4)\right\}.
\end{aligned}
$$

Higher order phasing is equivalent to inclusion of higher order gravitational radiation reaction (GRR). 3.5PN (v^7/c^7) flux beyond leading quadrupole determines 3.5PN (v^7/c^7) Radiation Reaction (RR relative to leading RR at 2.5PN (v^5/c^5). Very recently 4PN terms in conserved energy for the quasi circular case have been obtained by ADM methods (Damour, Jaranowski, Schäfer). The 3PN GW flux includes many non-linear effects like tails, non-linear memory and tail of tails that can be visualized as follows:



3. Quasi-eccentric case, GW Data Analysis Applications

During 1995 - 2014, the group around me at RRI worked systematically on extending the work on quasi-circular inspiral to the quasi-eccentric case thereby extending classic results of Peters and Mathews for eccentric binaries to 3PN (Gopakumar, Arun, Qusailah, Sinha, Mishra; Blanchet, Faye). Our exposure to algebraic computing at RRI started with a copy of Maple, Ravi Subrahmanyan shared with us on his return from USA. Phasing for the eccentric case needs in addition to 3PN conserved energy E, the 3PN conserved angular momentum J^i; in addition to the 3PN far zone flux of energy $\mathcal{L}$, the far zone flux of angular momentum $\mathcal{J}^i$. The secular evolution of the orbital elements requires one to average the fluxes over an orbit. To average the fluxes over the orbit, one needs a 3PN generalized quasi-Keplerian representation that was provided by the Jena group of Memmesheimer, Gopakumar and Schäfer. Using this and our fluxes, we have provided the secular evolution of orbital elements generalizing Peters-Mathews to 3PN. We have further improved on this and gone beyond secular evolution by method of variation of constants (Damour, Gopakumar, Iyer, Konigdorffer).

4. Applications to Detection and Parameter estimation

In a series of publications, Damour and Sathyaprakash and I developed tools (Effectualness, Faithfulness, Window Functions, Inequivalent PN families, Effective one body methods) to deal quantitatively with template construction and understand template characterisation issues in GW data analysis (GWDA). In collaboration with Sathyaprakash, students in my group (Arun, Sundarrajan, Sinha, Mishra) investigated implications for parameter estimation, tests of gravity, implications of full waveform for astrophysics and cosmology for LISA and Einstein Telescope .

D. Status of the MPM-PN formalism

Work is in progress to obtain the 3.5PN GW polarizations. MathTensor was found to be inefficient in dealing with higher order multipoles and it was decided to use xTensor, which being more efficient in dealing with symmetries, was more efficient for higher order multipoles. Guillaume Faye rewrote from scratch the codes for xTensor and we validated it with the older MathTensor calculations up to 3PN. We now have a more efficient set of programs which were used to compute the 3.5PN mass quadrupole and 3PN mass octupole and the associated h^{22}, h^{33} and h^{31} to 3.5PN accuracy (Faye, Marsat, Blanchet, Iyer).

The Multipolar Post Minkowskian (MPM) formalism matching to a PN source is a good example of the advantage that a complete and mathematically rigorous treatment of a problem can eventually bring in the future for more demanding applications that could be around the corner. Currently it is the most successful since it can deal with *all* aspects of the required computations: the conservative EOM, radiation field at infinity, non-linear efffects related to tails, tails of tails and non-linear memory. It has evolved over the last two decades into a consistent algorithmic approach to analytical GW computations. For technical details see the excellent Living Review by Luc Blanchet and the comprehensive book on Gravitational Waves by M. Maggiore.

E. Other approaches

The computations of EOM are very efficiently implemented in the ADM approach (Damour, Schäfer, Jaranowski) who have been the first to obtain the 4PN conserved energy for ICB in quasi-circular orbits. Direct Integration of Relaxed Einstein Eqns - DIRE (Epstein, Thorne, Will and Wiseman, Pati); can be explicitly implemented to obtain the 2PN EOM and 2PN fluxes. Though one can use it to compute the 3.5PN terms in the EOM it has not yet been completed at 3PN. The Strong field point particle limit (Schutz, Futamase, Asada, Itoh) works well for the EOM and has provided the 3PN EOM. The effective field theory technique (Goldberger, Porto, Rothstein, Foffa, Sturani, Galley..) is a powerful approach that in principle is as complete as the MPM-PN. It is fair to say that up till now it has not been able to provide new results beyond the MPM-PN approach. The BH perturbation approach has yielded very high order results in the test particle limit both in the Schwarzschild and Kerr cases as also self-force approaches for extreme mass ratio inspirals (EMRI). Self-force PN comparisons (Blanchet, Tiec, Whiting, Detweiler, Johnson-McDaniel..) and Numerical Relativity - Analytical Relativity (NR-AR) comparisons (Cornell-Caltech, Goddard, Jena, RIT, IHES, Maryland) have provided very useful results over the last few years. Though ab initio computations are needed to compute the complete EOM, one can reconstruct partially the radiation reaction terms in the acceleration from balance equations for energy and angular momentum and expressions for the corresponding energy and angular momentum fluxes (Iyer, Will, Gopakumar, Sai Iyer ....)

F. Extending domains of PN expansions

The PN expansion breaks down as one approaches the last stable orbit (LSO) and we explored application of *Resummation methods* e.g Padé approximants (Damour, Iyer, Sathyaprakash (1997) to extend numerical validity of PN expansions (at least) up to the LSO . Effective-One-Body (EOB) approach is a new resummation, to extend validity of suitably resummed PN results beyond the LSO, and up to the merger (Buonanno and Damour 1998, 2000; Jaranowski, Schäfer, Damour and Nagar, Buonanno et al) Starting from Hamiltonian describing the conservative motion of two bodies and going to center of mass one obtains a PN expanded Hamiltonian describing relative motion. At Newtonian approximation, $H_0(\boldsymbol{q},\boldsymbol{p})$ can be thought of as describing a 'test particle' of mass μ orbiting around an 'external mass' GM. EOB approach is *general relativistic generalization* of this. Consists in looking for an 'external spacetime geometry' $g^{\rm ext}_{\mu\nu}(x^\lambda;GM)$ such that dynamics of 'test particle' of mass μ within $g^{\rm ext}_{\mu\nu}(x^\lambda,GM)$ is *equivalent* (when expanded in powers of $1/c^2$) to original, relative PN-expanded dynamics). The *four* essential elements of the EOB approach are: (i) *Hamiltonian* $H_{\rm real}$ describing *conservative* part of relative dynamics of 2 BH; (ii) *Radiation-reaction force* $\mathcal{F}_\varphi$ describing loss of (mechanical) angular momentum, and energy, of binary system which can be computed using (iii) and (iv) that follow; (iii) Definition of various multipolar components of *"inspiral-plus-plunge" (metric) waveform* $h^{\rm insplunge}_{\ell m}$; (iv) Attachment of subsequent *"Ringdown waveform"* $h^{\rm ringdown}_{\ell m}$ around certain (EOB-determined)"merger time" t_m. The EOB analytically provides complete GW signal emitted by inspiralling, plunging, merging and ringing binary black holes. In it can be included parameters characterising physical effects beyond what is currently computed (e.g. higher PN order terms in the equation of motion, next-to-circular effects, higher order quasi-normal modes) and by variation or *flexing* of these parameters the EOB can be further improved and calibrated to Numerical Relativity simulations. In 2008 Damour, Iyer and Nagar provided improved resummed templates for matching analytical results to exciting numerical relativity simulations of binary black hole merger. (It used as one of the inputs: extension of my 1990 work on current moments with Thibault.) The improved EOB models are based on a *multiplicative* decomposition of the multipolar waveform into a *product* of the Newtonian waveform and a PN-correction factor which is a *product of four factors.* The choice of factors, based on a physical understanding of the main effects influencing the final waveform, facilitates a graded improvement of the analytical waveform and possibility of refinement by match to improved numerical relativity results. Improved 5.5 PN GW polarizations in test particle limit were provided by Fujita and Iyer.

G. Looking back.. What worked..What did not

Activity on the perturbation approach to GW did not take off in India. For many years, I tried to seed Numerical Relativity in India via opportunities in NR groups abroad to Sai Iyer, Anshu Gupta, Gopakumar, Shrirang Deshingkar and Aseem Rastogi. For a different reason in each case, it did not take off. I still hope that Vijay Varma who joined Caltech grad school last year may opt for NR work and come back. To make an impact, one needs to catch the field about 3-5 years before it becomes hot.. One needs to have someone actively working here full time to seed an activity in a new area. Both Sanjeev and I got on the GWDA and GW source modelling at the right time.. Similarly too IndIGO's recent attempt to jump start GW Experiments in India. LIGO India proposal (more in later sections) would not have happened in 2011, had we not reacted earlier to Rana's suggestion in 2007 at the ICGC meeting in IUCAA on thinking about an IndIGO detector or responding to Blair's suggestion in 2008 for Indo-Australian collaboration on GW Astronomy when he contacted me on suggestion from Ravi Subrahmanyan and Lakshmi Sarapalli in a meeting in Australia. Needless to add these are `necessary` but not **sufficient**!! I have fallen out of sync with exciting work on self force, 4PN, extension to the spinning case. Both due to time spent on IndIGO and due to inability to invest quality time in critically keeping up with newer developments.

H. Concluding Remarks on I Phase

An experiment is driving the theory. The experimental challenge to detect GW has led to a remarkable progress in the theoretical two body problem in GR. The important insight of Thibault Damour in 300 years of Gravitation (1987) is equally relevant at this Centenary of GR:

`It is not sufficient to transplant in Einstein's theory the technical steps of Newton's theory but one needs to transmute within Einstein's conceptual framework the ideas that underlie the technical developments`

Indeed: Bliss was it in that dawn to be alive, ...

In 2009 began what became Phase II of my involvement with GW: the need to chip in to locate the elusive gravitational wave sources.. A task that continues till today..

V. TOWARDS GW DETECTION..

GW detectors today are laser interferometers. To achieve the sensitivity required one uses a power recycled Michelson interferometer with Fabry Perot arm cavities. By use of photo-detectors one can sense 10^{-11} of fringe. In the absence of a GW signal, the experiment is set at the dark fringe, so that there is no light at the output of the photo-diode. When the GW signal passes through, the arm length changes and you get some light at the photo diode indicating the presence of a possible GW signal. The interferometer acts as a *transducer* converting GW to a photocurrent proportional to the strain amplitude.

It is interesting to recall the assessment of the First Detection in the LIGO proposal. In the LIGO Proposal of 1987 for design and prototyping prior to the next construction proposal, it said

`There are non-negligible possibilities for wave detection with the first detector in the LIGO. Detection is probable at the sensitivity level of the advanced detector The first detection is most likely to occur, not in the initial detector in the LIGO but rather in a subsequent one, as the sensitivity and frequency are being pushed downward from the middle curve towards the bottom curve of`

The 4 km long initial LIGO detectors achieved $\Delta L \simeq 10^{-19} m/\sqrt{Hz}$. It is worth pointing outi for the subsequent narrative that LIGO consisted of two observatories with three detectors since at LIGO, Hanford there were two co-located interferometers in the same beam tube.

A. Gravitational wave legacy in India (1990 ..)

The Indian contribution to the global effort for detecting GW in this period was on two significant fronts over two decades: (i) Source Modeling at RRI in a group around me in close collaboration with French and Cardiff groups, (ii) GW Data Analysis at IUCAA in a group around Sanjeev Dhurandhar. This group collaborated with most international GW groups and was a LSC member for a decade since 2000. Unfortunately, in the same time frame, GW Experimental Proposals (IUCAA, CAT) though attempted did not take off. Fortunately, students and post-docs from the GW groups at IUCAA and RRI, after post-doc stints in leading GW groups abroad are currently back in faculty positions at different institutions in India.

B. The coming Advanced Detector decade

The coming decade will see progress towards the second generation or Advanced detectors with anticipated design displacement sensitivity $\Delta L \simeq 10^{-20} m/\sqrt{Hz}$. Since GW detectors are not flux but amplitude detectors, a ten fold increase in sensitivity corresponds roughly to a 1000 fold increase in the number of GW sources. It is expected that the advanced detectors would see as many as 40 NS binaries, 10 NS-BH binaries and 20 BH-BH binaries per year. Uncertainities in the astrophysics involved could spread these rates to (0.4-400) for NS-NS binaries, (0.2-300) for the NS-BH case and (2-400) for BH.BH binaries. It is not surprising that we have not detected any GW event with Initial LIGO. The expected rate is 1 NS-NS per galaxy per 10,000 years and initial ligo could see about 100 galaxies. However, it would be very surprising if we did `Not` detect anything with Advanced LIGO!!! Being able to see about 100, 000 galaxies, one could almost have 1 event per month!!

Where are we today in regard to aLIGO? Livingston L1 achieved a 2 hour stable lock in May 2014 and in March 2015 had a range of 67 Mpc for NS-NS binaries. Hanford H1 achived a 2 hour lock in Feb 2015 and has a range of 13 Mpc for the same. The two detectors are under commissioning now.. The first joint observing runs O1 of aLIGO is scheduled for the latter half of 2015. While first detections are indeed *possible* by the end of this year, detections are `likely` three or four years from now. When detections become routine one will progress to questions like: What are the sources of GW? and Where are these GW coming from? To the stirrings of GW Astronomy and the inauguration of multi-messenger astronomy including a GW window. For instance, the observation of GW coincident with GRBs will aid determination of progenitor. GW signal from core-collapse supernova may carry info about the supernova engine. GW and EM observations of binary coalescence will provide independent measures of distance and red-shift and hence enable precision tests of cosmology. Multi-messenger observations involve an accurate and rapid localization of the source by GW observations and the subsequent deployment of EM observations to follow up the event. Ambitious projects are envisaged to follow up GW candidates in a host of EM telescopes like the Palomar Transient factory, Pan Starrs, Sky Mapper, LOFAR. Their typical field of view is about 10 sq.deg and thus one would like the GW network to provide localization of GW sources on this scale. Observations like these would eventually throw light on equation of state (EOS) of neutron stars, nature of black holes in general relativity and tests of gravity beyond general relativity.

C. Sky Localization with GW detectors

GW detectors are nearly omni-directional and individually they provide almost no directional information. Only by an array working together can we determine source location. This is analogous to aperture synthesis in radio astronomy and the accuracy of localization is tied to diffraction limit of the detector. GW sources are polarized. Hence one needs complete polarization information to extract distances, energies, and other details of sources. GW detectors are polarization selective and can be completely insensitive to one linear polarization depending on their orientation. Thus one must have a three dimensional array of detectors to extract maximum science from the GW detector network.

Source localization is determined by triangulation using measured delays of arrival time of GW at different sites. Errors on time of arrival determine the sky location uncertainty. With two sites and one time delay the source could be anywhere on a localized ring in sky. With three Sites and two Time delays the sources are localized to two regions which are mirror images in detector plane. The localization is poor for sources in and close to this detector plane.

D. From IndIGO to LIGO-India

With the first generation detectors achieving design sensitivity, it was opportune to jump start Indian participation in GW Experiment and Science before it fully bloomed and it was too late. This led to the formation of the IndIGO Consortium in 2009. The IndIGO consortium explored participation in the Australian AIGO and later in LIGO-Australia and this eventually led to the LIGO-India opportunity. In June 2011 began the preliminary discussions for relocating the H2 Advanced LIGO interferometer in India. In July 2011, IndIGO was accepted as member of Gravitational Wave International Committee (GWIC) and in September 2011 as a member group of LIGO Scientific Collaboration (LSC). The formal offer by LIGO-Lab to collaborate on LIGO-India was in October 2011. Immediately after, in November 2011, a proposal was submitted to DAE & DST and presented at the DST meeting on Mega Projects. By December 2011 IPR, IUCAA and RRCAT were identified as lead institutions for LIGO-India.

E. LIGO-India: Why is it important?

The geographical relocation of the second Hanford detector is strategic for GW astronomy. A fourth site not in the plane formed by the two US LIGO sites and Virgo and far from them greatly improves source localization ability of the network. Any additional detector in the network leads to increased event rates (2-4 times by coherent analysis), improved duty cycle, improved detection confidence, improved sky coverage, and improved determination of the two GW polarizations. What is crucial is the improved source location required for multi-messenger astronomy. Inclusion of LIGO-India improves angular resolution on an average by four times and in some directions by a factor of 10-20 (Fairhurst, Sathyaprakash, Klimenko and Vedavato).

F. LIGO-India Project

The LIGO-India project proposal is for the construction and operation of a Advanced LIGO Detector (displacement sensitivity: $4 \times 10^{-20} m/Hz$) in India in collaboration with the LIGO Lab. The plan is to set up the Indian node of the three node global Advanced LIGO detector network by 2022 and operate it for 10 years. The entire hardware components of the aLIGO detector along with designs and software is to be provided by LIGO-USA and its UK, German and Australian partners ($120 M including R&D). The entire infrastructure including the 8 km uhv system, corner and end stations, related labs and clean rooms as well as the team to build and operate the Observatory will be the Indian responsibility ($250 M, 15 yrs).

G. Towards LIGO-India

After four LIGO-Lab visits to Indian institutions to assess scientific interest, technical and management capabilities (August, October, December 2011, March 2012) and three NSF reviews (October 2011, April and June 2012), in April 2012, LIGO-India was included as a DAE Mega Project in the Atomic Energy Commission (AEC) meeting. In August 2012 the National Science Board, USA, approved the proposed Advanced LIGO Project change in scope, enabling plans for the relocation of an advanced detector to India. In December 2012, LIGO-India was included and figured in the list of Mega-Projects in the report of National Development Council to approve the Twelfth Five year Plan. LIGO-India is a Mega Project on Indian soil. The final decision on its in principle approval by the cabinet is awaited.

LIGO-India is a project under Indo-US collaboration involving as funding agencies the NSF on the US side and jointly DAE(India) & DST(India) on the Indian side. At the level of institutions it involves the LIGO Laboratories at Caltech & MIT on the American side and three lead institutes Inter-University for Centre for Astronomy & Astrophysics (IUCAA), Institute of Plasma Research (IPR), and Raja Ramana Centre for Advanced Technology (RRCAT) on the Indian side. Broadly IUCAA would be responsible for site selection and survey, data analysis & computing facility, and science & human resource development; IPR for civil infrastructure and facilities, vacuum system & mechanical engineering and implementation of CDS system; and RRCAT for detector hardware documentation & pre-installation, optics & 3rd generation R&D detector integration, installation and commissioning. These activities will be executed under an MOU among the lead institutes. Scientific participation in LSC is via IndIGO-LSC which would be managed jointly by IUCAA and IndIGO under a MoU.

IndIGO is a multi-institutional, multi-disciplinary consortium consisting of the three lead institutions (IUCAA, IPR, RRCAT), nine nodal institutions (CMI, ICTS-TIFR, IISER-Pune, IISER-Kolkata, IISER-Tvm, IIT-Madras, IIT-Kanpur, IIT-Gandhinagar) and individual scientists from six other institutions. Starting with about ten members in 2009 and dominated by theorists and data analysts it currently has about one hundred and ten members with about 50% experimenters. A sub-group, IndIGO-LSC, is the Indian presence in LIGO Scientific Collaboration. A single group at IUCAA during 2000-2010 has grown currently to a nine institution (IPR, IUCAA, RRCAT, TIFR, ICTS-TIFR, CMI, IIT-Gn, IISER-Kolkata, IISER-Tvm) pan-Indian group with fifty one members.

H. Concluding Remarks - Phase II

GW detection is an audacious endeavor that strains all resources: It needs the best in technology: vacuum, seismic isolation, lasers, optics, controls.. The best theoretical templates, data analysis, computing, archiving .. It is nearly impossible with insurmountable challenges.. And that is why it is worth doing! It stresses the symbiotic relation between basic sciences and applied technology on one hand and theory, experiment and computation on the other..

The successful operation of Advanced Detectors is expected to transform the field from GW Detection to GW Astronomy by 2022 when the global GW detector network expands to include KAGRA in Japan and LIGO-India. It is important to emphasize that not only has LIGO-India the potential to be a critical element of a transformative astronomy but more importantly LIGO-India will impact high precision experiments and high end technology in India

Four centuries after Galileos telescope launched optical astronomy and a century after Einsteins inspired discovery of general relativity, a major revolution in astronomy is round the corner with a facility in India having the opportunity to play a key role. LIGO-India will be a critical element of the global GW detector network for GW astronomy. Though most of the Universe is dark in EM, we can reconstruct it from the footfalls of Einsteins messengers that will be heard in our GW detectors in the near future. Today, though Einstein seems to be right, the question remains: Is he 100 % right?? Many enigmatic aspects of gravity still remain Decoding gravitation is the holy grail and the Odyssey encompasses the whole universe most of it dark with its symphony in GW from the chirps and wails of dying stars and black holes and eventually the murmurs of the big bang .. It is exciting that in this *Year of Light* and `Centenary of GR` the approval for LIGO-India is imminent.. So with an invitation to explore the rich GW spectrum, it is fair to say
`If Bliss was it in that Dawn to be alive.. To be young in the coming decades would be the very heaven..`
So Do ALL that you can to realize this dream!!

VI. FURTHER READING

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