Absence of Horizon in the Gravitational Collapse with Radiation

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Joshi and his collaborators, [Joshi and Goswami (2007)] recently constructed a family of solutions of Einstein's field equations which represented perfect fluid undergoing gravitational collapse. At the beginning the fluid satisfied all the energy conditions, while at the late stage the pressure was allowed to be negative. The weak energy condition was, however, still satisfied. There was a massive ejection of matter in the late stage of collapse and hence could avoid the occurrence of an event horizon.In what follows there is another approach to explain the mass loss from the collapsing sphere. It is the dissipation of heat from the interior of the collapsing body. The interior is matched with the exterior Vaidya's radiating metric. In this model the mass loss due to radiation is accompanied with the decrease of the boundary radius in such a manner that the horizon does not appear on the surface of the collapsing sphere at any stage.

I. FLUID SPHERE WITH HEAT FLOW AND THE JUNCTION CONDITION

The time-like three surface divides space-time into the interior and the exterior manifolds V^- and V^+.

$$dS^2_{(-)} = -e^{\nu(r,t)}dt^2 + e^{\lambda(r,t)}(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2)$$

$$dS^2_{(+)} = -\left(1 - \frac{2M(v)}{\bar{r}}\right) dv^2 - 2d\bar{r}dv + \bar{r}^2(d\theta^2 + sin^2\theta d\phi^2)$$

On the boundary surface Σ we have

$$dS^2_{\Sigma} = -d\tau^2 + A^2(\tau)(d\theta^2 + sin^2\theta d\phi^2)$$

The energy momentum tensor in the interior of a rotation free and shear free fluid with heat flow is

$$T^{\mu\nu} = (\rho + p)v^\mu v^\nu + g^{\mu\nu}p + q^\mu v^\nu + q^\nu v^\mu \ ,$$

q^μ is the heat flux defined as $q^\mu = q(t,r)\delta^r_\mu$ and $q^\mu v_\mu = 0$.

Continuity of the first fundamental form leads to

$$dS^2_{(-)} = dS^2_{(+)} = dS^2_{(\Sigma)}$$

at the boundary.

Using the Israel-Darmois [1996] boundary conditions for the continuity of the second fundamental form

$$k^{(+)}_{ij} - k^{(-)}_{ij} = 0.$$

The extrinsic curvature forms on the surface

$$k^{(\pm)}_{ij} = -n^{(\pm)}_\mu \frac{\partial^2 x^{\mu(\pm)}}{\partial\xi^i \partial\xi^j} - n^{(\pm)}_\mu \Gamma^{\mu(\pm)}_{\gamma\delta} \frac{\partial x^{\gamma(\pm)}}{\partial\xi^i} \frac{\partial x^{\delta(\pm)}}{\partial\xi^j}$$

ξ^i's are coordinate on the boundary hyper surface. $n^{\mu(\mp)}$ are the unit normal vectors on Σ in the interior and the exterior regions.

In terms of the interior co-moving coordinates

$$r - r_\Sigma = 0,$$

where r_Σ=constant.

$$n_\mu{}^{(-)} = (0, e^{\frac{\lambda}{2}}, 0, 0)$$

In terms of the exterior Vaidya-coordinates the boundary is

$$\bar{r} - \bar{r}_{\Sigma}(v) = 0 \quad and$$

$$n_{\mu}{}^{(+)} = \left(1 - \frac{2M(v)}{\bar{r}} + 2\frac{d\bar{r}}{dv}\right)^{\frac{-1}{2}} (-\frac{d\bar{r}}{dv}, 1, 0, 0)$$

Continuity of the first fundamental forms:

$$(e^{\frac{\lambda}{2}} r)_{\Sigma} = A_{\Sigma}(\tau) = \bar{r}_{\Sigma} \tag{1}$$

$$(e^{\frac{\nu}{2}} \frac{dt}{d\tau})_{\Sigma} = 1 \tag{2}$$

$$(\frac{d\tau}{dv})^{2}_{\Sigma} = \left(1 - \frac{2M(v)}{\bar{r}} + 2\frac{d\bar{r}}{dv}\right)_{\Sigma} \tag{3}$$

Extrinsic curvature components:

$$k^{-}_{\theta\theta} = cosec^{2}\theta k^{-}_{\phi\phi} = \left[r(e^{\frac{\lambda}{2}} r)'\right]_{\Sigma} \tag{4}$$

$$k^{+}_{\theta\theta} = cosec^{2}\theta k^{+}_{\phi\phi} = \left[\bar{r}(\frac{d\bar{r}}{d\tau}) + \bar{r}(1 - \frac{2M(v)}{\bar{r}})\frac{dv}{d\tau}\right]_{\Sigma} \tag{5}$$

$$k^{-}_{\tau\tau} = (e^{\frac{-\lambda}{2}} \frac{\nu'}{2})_{\Sigma} \tag{6}$$

$$k^{+}_{\tau\tau} = \left[\frac{d\bar{r}}{d\tau}(\frac{d^{2}v}{d\tau^{2}}) - \frac{d^{2}\bar{r}}{d\tau^{2}}(\frac{dv}{d\tau}) - \frac{3M(v)}{\bar{r}^{2}}\frac{d\bar{r}}{d\tau}(\frac{dv}{d\tau})^{2} + \frac{1}{\bar{r}}\frac{dM}{d\tau}(\frac{dv}{d\tau})^{2} - (1 - \frac{2M(v)}{\bar{r}})\frac{M}{\bar{r}^{2}}(\frac{dv}{d\tau})^{3}\right]_{\Sigma} \tag{7}$$

Total mass contained within the co moving radius 'r' is given by Cahil and McVittie(1970)

$$m(r,t) = \frac{1}{2}(e^{\frac{\lambda}{2}} r)[e^{-\nu}[(e^{\frac{\lambda}{2}} r)^{\cdot}]^{2} - e^{-\lambda}[(e^{\frac{\lambda}{2}} r)']^{2} + 1] \tag{8}$$

In view of (8)

$$k^{-}_{\theta\theta} = \bar{r}_{\Sigma}[1 - \frac{2m}{\bar{r}} + U^{2}]^{\frac{1}{2}}_{\Sigma},$$

$$where\ U_{\Sigma} \equiv (\frac{d\bar{r}}{d\tau})_{\Sigma} = [e^{\frac{-\nu}{2}} \frac{d}{dt}(e^{\frac{\lambda}{2}} r)]_{\Sigma}$$

$$k^{+}_{\theta\theta} = \bar{r}_{\Sigma}[1 - \frac{2M}{\bar{r}} + U^{2}]^{\frac{1}{2}}_{\Sigma},$$

Equating

$$m_\Sigma = M \tag{9}$$

Using Einstein's field equation $G^\mu_\nu = +8\pi T^\mu_\nu$ in the relation

$$k^{(-)}_{\tau\tau} = k^{(+)}_{\tau\tau}$$

We obtain

$$[T^{1(-)}_1 + e^{\frac{(\lambda-\nu)}{2}} T^{1(-)}_0]_\Sigma = 0 \tag{10}$$

This is a general result valid for a fluid admitting viscosity also [Banerjee and Dutta Choudhury (1989)].

For a non-viscous fluid (10) leads to $p = (q^1 q_1)^{\frac{1}{2}}$ at the boundary. If there is no heat flux the pressure at the boundary $p_\Sigma = 0$. The exterior is then Schwarzschild space time $M = constant$. For an observer at rest at infinity, the total luminosity is given by

$$L_\infty(v) = 4\pi r^2 q = -\frac{dM}{dv} \tag{11}$$

q is the energy flux density of the radiation and at the exterior

$$T_{\mu\nu} = -\frac{1}{4\pi r^2}\frac{dM}{dv}\delta^0_\mu \delta^0_\nu$$

II. FLUID SPHERES WITH HEAT FLUX AND ISOTROPIC PRESSURE:

We use different symbols

$$A^2 = e^\nu \quad and \quad B^2 = e^\lambda$$

The pressure isotropy leads to

$$\frac{A_{xx}}{A} - \frac{F_{xx}}{F} + 2\frac{A_x}{A}.\frac{F_x}{F} = 0$$

where $F = \frac{1}{B}$ and $x = r^2$.
(1) Assume $F_{xx} = 0$

$$B = \frac{1}{a(t)r^2 + b(t)} \ , \quad A = \frac{c(t)r^2 + d(t)}{a(t)r^2 + b(t)}$$

These solutions are conformally flat (Weyl curvature terms vanish) [Banerjee and Som (1981)]. Suitable choices of $a(t), b(t), c(t)$ *and* $d(t)$ gives Maiti's (1982) form.

$$dS^2 = -[1 + \frac{\eta(t)}{1+\xi(t)r^2}]^2 dt^2 + \frac{R^2(t)}{(1+\xi(t)r^2)}(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2)$$

(2) Assume $a(t) = 0$, $b(t) = d(t)$
The metric reduces to

$$dS^2 = -(1 + \xi(t)r^2)^2 dt^2 + R^2(t)(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2) \quad [Modak(1984)].$$

A special class of Modak's solution $\xi(t) = \xi_0$, which is a constant.

$$A = (1 + \xi_0 r^2) \quad and \quad B = R(t).$$

The mass parameter of the distribution upto the boundary following from (8) is given by [9]

$$2m_\Sigma = \left[\frac{r^3 B\dot{B}^2}{A^2} - 2r^2 B' - \frac{r^3 B'^2}{B}\right]_\Sigma = \frac{r_0^3 R\dot{R}^2}{(1+\xi_0 r_0^2)} \tag{12}$$

and $\bar{r}_\Sigma = R(t) r_0$,

where r_0 =boundary radius in comoving radial coordinate.

The boundary condition [Santos(1985), Bonnor et al(1989)] $p_\Sigma = (q_1 q^1)^{\frac{1}{2}}_\Sigma$ yields

$$2R\ddot{R} + \dot{R}^2 + C_1\dot{R} = C_2 \tag{13}$$

where C_1 and C_2 are constants.
A simple solution of (13)

$$R(t) = -ct \tag{14}$$

where c is a constant.

A very interesting result from this solution

$$\frac{2m_\Sigma}{\bar{r}_\Sigma} = \frac{2m_\Sigma}{R(t) r_\Sigma} = \frac{r_0^2 c^2}{(1+\xi_0 r_0^2)} \tag{15}$$

The right hand side is independent of time. Once it is chosen less than unity the quantity $\left(1 - \frac{2M(v)}{\bar{r}_\Sigma}\right)$ remains always positive and hence the horizon never appears at the boundary.

This happens because here the rate of mass loss in the form of radiation is the same as the rate of fall of the boundary radius in the collapse.

III. PROPERTIES OF THE SOLUTION:

In the equation (13)

$$C_1 = -4\xi_0 r_0\ , \quad C_2 = 4\xi_0(1+\xi_0 r_0^2)$$

so that

$$c = \frac{1}{2}[-|C_1| + (C_1^2 + 4C_2)^{\frac{1}{2}}]$$

Hence $c > 0$.
The solution $R(t) = -ct$ presents collapse for $-\infty < t \leq 0$.

Density, pressure and heat flow vector are

$$\rho = \frac{3}{t^2(1+\xi_0 r^2)^2} \tag{16}$$

$$p = \frac{1}{t^2(1+\xi_0 r^2)^2}\left[\frac{4\xi_0}{c^2}(1+\xi_0 r^2) - 1\right] \tag{17}$$

and

$$q^1 = + \frac{4\xi_0 r}{c^2 t^3 (1+\xi_0 r^2)^2} \tag{18}$$

$\xi_0 = 0$ corresponds to no heat flux. The solution reduces to FRW space-time.

All these quantities diverge at $t \rightarrow 0$, the final step of the collapse.
There is no particular equation of state. In the interior all the energy conditions are satisfied.
Investigations of the expressions for ρ and p reveal that $\rho > 0,\ p > 0,\ \rho' < 0$. Further $p' < 0$ at $r = 0$ would be satisfied provided the following condition is satisfied

$$c^2 < 2\xi_0.$$

In addition the condition $\rho > p$ requires

$$c^2 > \xi_0(1 + \xi_0 r_0^2)$$

So finally a physically realistic model would require the inequality relation

$$2\xi_0 > c^2 > \xi_0(1 + \xi_0 r_0^2)$$

One of the consequences of this relation is $(1 - \xi_0 r_0^2) > 0$.
Since the fluid is conducting heat it must satisfy another condition such as

$$(\rho + p) > 2|q|, \quad where\ |q| = (g_{\mu\nu} q^\mu q^\nu)^{\frac{1}{2}} \tag{19}$$

in order to be consistent with all the energy conditions [Kolassis, Santos and Tsaubellis(1988)]. The condition (19) demands

$$1 + \frac{2\xi_0}{c^2}(1 + \xi_0 r^2) > \frac{4\xi_0 r}{c},$$

which may also be written in a different form

$$(1 - \frac{2\xi_0 r}{c})^2 > -\frac{2\xi_0}{c^2}(1 - \xi_0 r^2). \tag{20}$$

Naidu and Govender(2007) studied our model in the context of casual heat transport equation. In particular they investigated the relaxational effects on the temperature profile within the frame work of truncated Israel-Stewart transport equation

$$\tau h_\mu^\nu \dot{q}_\nu + q_\mu = -k(D_\mu T + T\dot{v}_\mu),$$

where τ is the relaxation time for the thermal signals and k is the thermal conductivity of fluid. Setting $\tau = 0$ the so called Eckart transport equation is regained. In this work Naidu and Govender have shown that the solutions for the temperature profile are such that the causal temperature exactly coincides with the noncausal temperature at the boundary $r = r_0$.

Extension of the result in more than four dimensional space time [Banerjee and Chatterjee (2005)].

In $(n + 2)$ dimensions the interior metric is

$$dS^2 = -A^2(t,r)dt^2 + B^2(t,r)(dr^2 + r^2 dX_n^2)$$

where $dX_n^2 = d\theta_1^2 + sin^2\theta_1 d\theta_2^2 + ...sin^2\theta_1 sin^2\theta_2 ...sin^2\theta_n d\theta_n^2$ The exterior generalized Vaidya metric

$$dS^2 = -(1 - \frac{2m(v)}{(n-1)\bar{r}^{n-1}})dv^2 - 2d\bar{r}dv + \bar{r}^2 dX_n^2$$

The isotropy of pressure leads to a special solution

$$dS^2 = -(1+\xi_0 r^2)dt^2 + R^2(t)(dr^2 + r^2 dX_n^2)$$

The boundary condition $p_\Sigma = (qB)_\Sigma$ yields

$$2R\ddot{R} + (n-1)\dot{R}^2 + N_1\dot{R} = N_2$$

N_1 *and* N_2 are constants. For a particular solution $R(t) = -ct$, the calculations are similar to those done in the earlier 4-dimensional case yield an equation modified for $(n+2)$ dimensions

$$\frac{2M_\Sigma}{(n-1)\bar{r}_\Sigma^{n-1}} = \frac{c^2 r_0^2}{(1+\xi_0 r_0^2)^2}$$

The right hand side is a constant. It does not even depend on the number of dimensions. One can choose the right hand side less than unity and hence the horizon does not appear. Now the density, heat flow and pressure are modified in higher dimensions

$$\rho = \frac{n(n+1)}{2t^2(1+\xi_0 r^2)^2}$$

$$p = \frac{n}{t^2}[\frac{2\xi_0}{2(1+\xi_0 r^2)^2} - \frac{(n-1)}{2(1+\xi_0 r^2)^2}]$$

$$q = -\frac{2n\xi_0 r}{c^2 t^3 (1+\xi_0 r^2)^2}$$

here also the energy conditions are satisfied. The above calculations are done for the completeness of our final result.

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